Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM -101	Course Title: **Differential Calculus**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Let f be defined on R such that $f(x) = 0 \quad and \quad f(x) = \frac{e^{1/x}}{1+e^{1/x}}\ when\ x \neq 0$

   Does $limt\ exit\ when\ x \to 0$

2. Let f be defined on R such that $f(x) = 5x - 4\ when\ 0 \leq x \leq 1$

   $$f(x) = 4x^2 - 3\ when\ 1 \leq x \leq 2$$

   $$f(x) = 5x + 4\ when\ x > 2$$

   $is\ f\ continuous\ at\ x = 1\ and\ x = 2$?

3. Show that if faction is differentiable at given point then it is continuous at that point. is the converse true ? Support your answer.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Let R be a relation defined in the set of natural numbers N such that

   $R = \{(x, y): 3x + y = 15\}$find the domain and range of R.

5. Let $f: \mathbb{R} \to \mathbb{R}$ be a map defined by $f(x) = x^2 \quad and$

   $$let\ A = \{x\epsilon\mathbb{R} : 1 \leq x \leq 2\}\ find\ f(A)$$

6. If $fx = 2x - 1\ \ and\ \ g(x) = x + 4\ then\ find\ (f.g)(x)$.

7. $Consider\ a\ map\ f: \mathbb{R} \to \mathbb{R}\ such\ that\ f(x) = 4x^2 - 3\ is\ f\ injective.$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-102	Course Title: **Analytical Geometry**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

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1. Find the point of intersection of the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{2}$ with the plain $3x + 4y + z = 10$
2. Find the equation of the sphere for which the circle
   $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0,\ 2x + 3y + 4z = 8$ is a great circle.
3. Find the equation of the tenant plains of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 30 = 0$ which are parallel $2x - y + z = 0$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. $If\ the\ equation\ x^2 - y^2 - 2x + 2y + \lambda = 0$
   $represent\ a\ degenerate\ conic\ then\ find\ the\ value\ of\ \lambda$
5. Find the angle between the pair of straight lines $x^2 + 4y^2 - 7xy = 0$
6. Find the perpendicular distance from the origin to the plain $x + 2y + z = 3$ also find the direction cosines of the normal to the plain.
7. Find the angle between the planes $2x - y + z = 5$ and $x + 3y + 2z = 7$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-103	Course Title**: Integral Calculus**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Show that $xy = 1$ *and* $x^2 + y^2 = 2$ *tuch each other at two points.*
2. Under what condition the curves $a_1x^2 + b_1y^2 = 1$ *and* $a_2x^2 + b_2y^2 = 1$ cut orthogonally
3. Find the angle of the intersection of the curves $y^2 = x$ *and* $x^2 + y^2 = 4$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Show that $\int_0^{\pi/2} (sin^2 x) \cos x \, dx = \frac{1}{3}$
5. Integrate $e^{ten\,x}.sec^2 x \quad w.r.t. \quad x$
6. Evaluate $\int_0^{\pi/4} (ten^5 x) dx$
7. Integrate $\frac{\sqrt{x}}{1+x^{1/4}} \quad w.r.t. \quad x$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-104	Course Title**: Differential Equation**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

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1. Solve that differential equation
   $(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2)\,dy = 0$
2. Solve $x^2 + p^2x = yp$
3. Find the orthogonal trajectories of the cardiod $r = a(1 - \cos\theta)$, a being the parameter.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Solve $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$
5. Solve $x.Dy + y = xy^3$
6. Solve $y = cx + {}^{a}/_{c}$
7. Is the following equation excel $\left(1 + e^{x/y}\right)dx + e^{x/y}\left(1 - {}^{x}/_{y}\right)dy = 0$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-105	Course Title**: Mechanics-I (Statics and Dynamics)**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If T be the tension at any point P of a common catenary and To be the tension at the lowest point A then prove that $T^2 - To^2 = W^2$ $when\ W\ in\ the$ weight of the are AP of the cetenery.
2. Five weight less rods of equal length are joined together so as to from a rhombus ABCD with one diagonal *BD*. at a weight W be attached to C and the system be suspended from A then show that there is a thrust in *BD* equal $w/\sqrt{3}$.
3. The velocities of a pastiche along and perpendicular to the radius vector from a fixed point are λr & $\mu\theta$. Find the path of the particle.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. A particle is allowed to move from the top of a cycloid whose vertex is upward and plane vertical with negligible velocity. Find the point where the particle leaves the cycloid.
5. A body consisting at a core and a hemisphere on the same base rests on a rough horizontal table the hemisphere being in contact with the table of the height of the cone is $\sqrt{3}$ times the radius of the hemisphere . Find whether the equilibrium will be stable or unstable.
6. A particle moves with a central acceleration which varies inversely as the cube of the distance if it is projected from an apse at a distance a from the origin with velocity which is $\sqrt{2}$ time of the velocity for a circle of radius a then show that its path is $r\ \cos\frac{\theta}{\sqrt{2}} = a.$
7. A particle whose mass is m is acted upon by a force $m\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin if it stats from rest a distance a then show that it will arrive at the origin is time $\frac{\pi}{4\sqrt{\mu}}$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-106	Course Title**: Mechanics-II (Dynamics and Hydrodynamics)**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the moment of inertia of a rod of length $2a$ & mass M about a line through its centre perpendicular to its length.
2. Find the moment of inertia of a circular disc of radian $'a'$ about its diametre.
3. At the vertex c of a tangle ABC which is a right angle at c show that the principle axis in the plane are inclined to the sides at an angle $\frac{1}{2}\ tan^{-1}\frac{ab}{a^2-b^2}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. One end of a light string is fixed to a point of the rim of a uniform circular disc of radian 'a' & mass 'm' and the string is wound several times round the rim. the free end is attached to a fixed point and the disc is held so that the part of the string not in contact with the vertical of the disc be let go find the acceleration & tension of the string.
5. Find the moment of inertia of a right circular cylinder about a straight line through its centre of gravity perpendicular to its axis.
6. A straight uniform rod can turn freely about one end O, hangs from O vertically. Find the least angular velocity with which it must begin to moves so that it may perform complete revolution in a vertical plane.
7. Show that the moment of inertia of the area bounded by $r^2 = a^2cos\ 2\theta$ about its axis is $\frac{Ma^2}{16}(\pi - 8/3)$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-107	Course Title**: Linear Algebra**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find all eign values and eign vectors of a linear transformation
   $T : IR^3 \; IR^3, \; defined\, as \;\; T\,(x, y, z) = (2x + y,\; y - z, 2y + 4z)$. Is T diagonolizatble
2. If w_1 and w_2 are any two finite subspaces of a vector space V then show that
   $$dim\,(w_1 + w_2) = dim\, w_1 + dim\, w_2 - dim\,(w_1 \cap w_2)$$
3. Find the eigen Values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Let V be a vector space over a field F such that it has no proper subspace. Then show that either
   $$V = \{\, o \,\}\; or\; dim\, V = 1.$$
5. Which of the following is a linear transformation where $T : IR^2 \rightarrow IR^2$
   (a) $T\,(x_1, x_2) = (1 + x_1, x_2)$
   (b) $T\,(x_1, x_2) = (x_2, x_1)$
6. A function f is defined on IR^2 as follows:
   $$f\,(x, y) = (x_1 - y_1)2 + x_1 y_2, where\; x = (x_1 - x_2)\; and\; y = (y_1, y_2)$$
   $$Is\; f\; a\; bilinear\; forms?\; Verify.$$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-108	Course Title**: Calculus of function of several variable and Vector Calculus**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. at $u = e^{xyz}$ *then show that* $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)$ is it also equal to $\frac{\partial^3 u}{\partial y \partial z \partial x}$?
2. Show that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$
3. A particle moves so that its position vector in given by $\bar{r} = \hat{\imath}\cos wt + \hat{\jmath}\sin wt$ Show that the velocity $\bar{v}$ is perpendicular $\bar{r}$ and $\bar{r} \times \bar{v}$ is constant vector.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Find the deviational derivative of $f(x) = \mathrm{xy}^2 + \mathrm{yz}^3$ at the point $(1, -1, 1)$ along the vector $\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$
5. at $\mathrm{u} = \tan^{-1}\left(\frac{\mathrm{x}^3+\mathrm{y}^3}{\mathrm{x}-\mathrm{y}}\right)$ then show that $\mathrm{x}\frac{\partial \mathrm{u}}{\partial \mathrm{x}} + \mathrm{y}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = \sin 2\mathrm{u}$
6. Determine the point where the function $\mathrm{x}^4 + \mathrm{y}^4 - 2\mathrm{x}^2 + 4\mathrm{xy} - 2\mathrm{y}^2$ has a maximum are minimum.
7. Find curl (curl $\bar{\mathrm{F}}$) at the point (0,1,2) where $\bar{\mathrm{F}} = (\mathrm{x}^2\mathrm{y})\hat{\imath} + (\mathrm{xyz})\hat{\jmath} + (\mathrm{z}^2\mathrm{y})\hat{\mathrm{k}}$

   Or

   Evaluate $\int \bar{F}.d\bar{r}$ *whre* $\bar{F} = (3x^2)\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the straight line joinery (0,0,0) & (2,1,3)

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: DCEMM-109	Course Title**: Abstract Algebra**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. State and Prove fundamental theorem of group homomorphism.
2. Let N be a normal subgroups of a group G and H be a subgroup of G then show that:

   (i) H ∩ N is normal subgroup of H (ii) HN is a subgroup of G (iii) N is normal subgroup of HN.
3. Prove that if G is abelian then G| Z(G) is cyclic where Z(G) is centre of G.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note: Answer each question in 200 to 300 Words. All carry equal marks.

4. Give all sub groups of (Z_{12}, +)
5. Let f: $G_1$1 → G_2 be a group homomorphism then show that kernel f is a normal subgroup of G_1.
6. Give an example non-cycle group whose all subgroups are cyclic.
7. Find all zero divisor elements of $Z/20$.

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: DCEMM-110	Course Title**: Number Theory**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the remainders obtained on division of the following:

(a) 3^{50} by 101 (b) 159^{7654} by 23

2. Find the g.c.d. of 163 and 34 and express it in the form $163m + 34n$ in two ways.

3. Prove that (a) $18! + 1 \equiv 0 \pmod{437}$ (b) $28! + 233 \equiv 0 \pmod{899}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Show that every square is congruent to 0 or 1 (mod 8).

5. Find the value of $\emptyset(m) if\ m = 500$.

6. Find the following Legendre symbols: (a) $\left(\frac{19}{41}\right)$ (b) $\left(\frac{3}{7}\right)$ (c) $\left(\frac{5}{11}\right)$ (d) $\left(\frac{6}{11}\right)$

7. Find the value of Mobius function $\mu(n)$ for n

(a) 15 (b) 30 (c) 47 (d) 100

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: DCEMM-112	Course Title: **Advance Analysis**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Every Cauchy sequence (S_n) of real Numbers converges.
2. Let (X_l, d_l) and (X_2, d_2) be two discrete metric spaces. Then verify that the product metric on X_1 x X_2 is discrete.
3. Show that a Cauchy sequence is convergent$\Leftrightarrow$ it has a convergent subsequence.
4. Let (X, d) be a metric space and $A \subseteq X$.Show that $\bar{A} = \{x \in X: d(x, A) = 0\}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. Define Complete Metric Space. Given an example of a metric space which is not Complete.

6. Any compact metric space is totally bounded.

7. Statement and Prove Mean value theorem.

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: DCEMM-113	Course Title**: Function of Complex Variable**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If $u = \frac{1}{2}\log(x^2 + y^2)$, find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z.
2. Find the radius of convergence R of the following power series:

   $(i)\ \sum_{n=0}^{\infty} z^n$ (ii) $\sum_{n=1}^{\infty} \frac{z^n}{n}$ (iii) $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$
3. Using Cauchy integral formula, calculate the following integrals.

   $\int_c \frac{\cos(\pi z)}{z(z^2+1)} dz$, where C is the circle $|z| = 2$
4. Evaluate $\int_0^{3+i} z^2 dz$ along the line joining the points (0, 0) and (3, 1).

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. Evaluate $\int_c \frac{dz}{z-2}\ for\ n = 2,3,4$ ...where $z = a$ is a point inside the simple closed curve c.

6. Find Taylor Series of $f(z) = \frac{1}{z}$ about $z = -1, z = 1$ and $z = 2$. Determine the circle of convergence in each case.

7. For the conformal transformation $w = z^{2.}$. Show that the circle $|z - 1| = 1$ transforms into the cardioid $R = 2(1 + cos\emptyset)$ where $w = Re^{i\theta}$ in the w −plane.

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: SBSMM-03	Course Title**: Elementary Analysis**	Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Write truth tables fo the sentence $\mathrm{P} \Rightarrow \mathrm{P}$ and

$\mathrm{P} \Rightarrow -\mathrm{P}$. Is the First sentence a tautology.

2. The diagonal or the equality relation & $in\ a\ set\ S\ is\ an\ equivalenc$e

relation in S. For it $\mathrm{x}, \mathrm{y} \in \mathrm{S}$ the x y iff $\mathrm{x} = \mathrm{y}$.

3. Let x be a set. Consider the relation R in $\big(\mathrm{e}(\mathrm{x})\big)$, given by : for A, B

$\in \big(\mathrm{e}(\mathrm{n})\big)$ARB if $\mathrm{A} \subseteq \mathrm{B}$.

4. $Let\ f: X \to Y\ be\ a\ map\ and\ let\ A\ and\ B\ subsets\ of\ X, then\ A \subseteq B \Rightarrow f(A)$

$\subseteq f(B)$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. $Let\ X = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y = [-1,1]$

$Let\ f: X \to Y\ given\ by\ f(x) = sinx, x \in X.$

6. Evaluate $\iint xy\, dxdy$ over the region in the positive quadrant for which $x + y \leq 1$.

7. Find the volume inside the paraboloid $x^2 + 4z^2 + 8y = 16$ and on the positive side of xz −plane.