

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM -101	Course Title: <b>Differential Calculus</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Let  $f$  be defined on  $\mathbb{R}$  such that  $f(x) = 0$  and  $f(x) = \frac{e^{1/x}}{1+e^{1/x}}$  when  $x \neq 0$   
Does *limit exist* when  $x \rightarrow 0$
2. Let  $f$  be defined on  $\mathbb{R}$  such that  $f(x) = 5x - 4$  when  $0 \leq x \leq 1$   
 $f(x) = 4x^2 - 3$  when  $1 \leq x \leq 2$   
 $f(x) = 5x + 4$  when  $x > 2$   
is  $f$  continuous at  $x = 1$  and  $x = 2$  ?
3. Show that if a function is differentiable at given point then it is continuous at that point. is the converse true ? Support your answer.

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Let  $R$  be a relation defined in the set of natural numbers  $\mathbb{N}$  such that  
 $R = \{(x, y): 3x + y = 15\}$  find the domain and range of  $R$ .
5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a map defined by  $f(x) = x^2$  and  
let  $A = \{x \in \mathbb{R} : 1 \leq x \leq 2\}$  find  $f(A)$
6. If  $fx = 2x - 1$  and  $g(x) = x + 4$  then find  $(f \cdot g)(x)$ .
7. Consider a map  $f: \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x) = 4x^2 - 3$  is  $f$  injective.

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-102	Course Title: <b>Analytical Geometry</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the point of intersection of the line  $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{2}$  with the plain  $3x + 4y + z = 10$
2. Find the equation of the sphere for which the circle  $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ ,  $2x + 3y + 4z = 8$  is a great circle.
3. Find the equation of the tenant plains of the sphere  $x^2 + y^2 + z^2 - 2x + 4y - 6z + 30 = 0$  which are parallel  $2x - y + z = 0$

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. *If the equation  $x^2 - y^2 - 2x + 2y + \lambda = 0$  represent a degenerate conic then find the value of  $\lambda$*
5. Find the angle between the pair of straight lines  $x^2 + 4y^2 - 7xy = 0$
6. Find the perpendicular distance from the origin to the plain  $x + 2y + z = 3$  also find the direction cosines of the normal to the plain.
7. Find the angle between the planes  $2x - y + z = 5$  and  $x + 3y + 2z = 7$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-103	Course Title: <b>Integral Calculus</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Show that  $xy = 1$  and  $x^2 + y^2 = 2$  touch each other at two points.
2. Under what condition the curves  $a_1x^2 + b_1y^2 = 1$  and  $a_2x^2 + b_2y^2 = 1$  cut orthogonally
3. Find the angle of the intersection of the curves  $y^2 = x$  and  $x^2 + y^2 = 4$

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Show that  $\int_0^{\pi/2} (\sin^2 x) \cos x \, dx = \frac{1}{3}$
5. Integrate  $e^{\tan x} \cdot \sec^2 x$  w.r.t.  $x$
6. Evaluate  $\int_0^{\pi/4} (\tan^5 x) \, dx$
7. Integrate  $\frac{\sqrt{x}}{1+x^{1/4}}$  w.r.t.  $x$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-104	Course Title: <b>Differential Equation</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Solve that differential equation

$$(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2) dy = 0$$

2. Solve  $x^2 + p^2x = yp$
3. Find the orthogonal trajectories of the cardioid  $r = a(1 - \cos \theta)$ ,  $a$  being the parameter.

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Solve  $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

5. Solve  $x.Dy + y = xy^3$

6. Solve  $y = cx + a/c$

7. Is the following equation exact  $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)dy = 0$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-105	Course Title: <b>Mechanics-I (Statics and Dynamics)</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If  $T$  be the tension at any point P of a common catenary and  $T_0$  be the tension at the lowest point A then prove that  $T^2 - T_0^2 = W^2$  when  $W$  is the weight of the arc AP of the catenary.
2. Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. A weight  $W$  is attached to C and the system is suspended from A then show that there is a thrust in BD equal to  $w/\sqrt{3}$ .
3. The velocities of a particle along and perpendicular to the radius vector from a fixed point are  $\propto r$  &  $\mu\theta$ . Find the path of the particle.

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. A particle is allowed to move from the top of a cycloid whose vertex is upward and plane vertical with negligible velocity. Find the point where the particle leaves the cycloid.
5. A body consisting of a cone and a hemisphere on the same base rests on a rough horizontal table the hemisphere being in contact with the table. The height of the cone is  $\sqrt{3}$  times the radius of the hemisphere. Find whether the equilibrium will be stable or unstable.
6. A particle moves with a central acceleration which varies inversely as the cube of the distance if it is projected from an apse at a distance  $a$  from the origin with velocity which is  $\sqrt{2}$  times the velocity for a circle of radius  $a$  then show that its path is  $r \cos \frac{\theta}{\sqrt{2}} = a$ .
7. A particle whose mass is  $m$  is acted upon by a force  $m\mu \left(x + \frac{a^4}{x^3}\right)$  towards the origin if it starts from rest at a distance  $a$  then show that it will arrive at the origin in time  $\frac{\pi}{4\sqrt{\mu}}$ .

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-106	Course Title: <b>Mechanics-II (Dynamics and Hydrodynamics)</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the moment of inertia of a rod of length  $2a$  & mass  $M$  about a line through its centre perpendicular to *its* length.
2. Find the moment of inertia of a circular disc of radius ' $a$ ' about *its* diameter.
3. At the vertex  $c$  of a triangle  $ABC$  which is a right angle at  $c$  show that the principal axis in the plane are inclined to the sides at an angle  $\frac{1}{2} \tan^{-1} \frac{ab}{a^2-b^2}$ .

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. One end of a light string is fixed to a point of the rim of a uniform circular disc of radius ' $a$ ' & mass ' $m$ ' and the string is wound several times round the rim. the free end is attached to a fixed point and the disc is held so that the part of the string not in contact with the vertical of the disc be let go find the acceleration & tension of the string.
5. Find the moment of inertia of a right circular cylinder about a straight line through its centre of gravity perpendicular to its axis.
6. A straight uniform rod can turn freely about one end  $O$ , hangs from  $O$  vertically. Find the least angular velocity with which it must begin to move so that it may perform complete revolution in a vertical plane.
7. Show that the moment of inertia of the area bounded by  $r^2 = a^2 \cos 2\theta$  about its axis is

$$\frac{Ma^2}{16}(\pi - 8/3)$$

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-107	Course Title: <b>Linear Algebra</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find all eigen values and eigen vectors of a linear transformation  
 $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ , defined as  $T(x, y, z) = (2x + y, y - z, 2y + 4z)$ . Is T diagonalizable
2. If  $w_1$  and  $w_2$  are any two finite subspaces of a vector space V then show that  
$$\dim(w_1 + w_2) = \dim w_1 + \dim w_2 - \dim(w_1 \cap w_2)$$
3. Find the eigen Values and eigen vectors of the matrix  $A = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Let V be a vector space over a field F such that it has no proper subspace. Then show that either  
 $V = \{0\}$  or  $\dim V = 1$ .
5. Which of the following is a linear transformation where  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$   
(a)  $T(x_1, x_2) = (1 + x_1, x_2)$   
(b)  $T(x_1, x_2) = (x_2, x_1)$
6. A function f is defined on  $\mathbb{R}^2$  as follows:  
 $f(x, y) = (x_1 - y_1)^2 + x_1 y_2$ , where  $x = (x_1 - x_2)$  and  $y = (y_1, y_2)$   
Is f a bilinear forms? Verify.

# Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2022-23

Course Code: UGMM-108	Course Title: <b>Calculus of function of several variable and Vector Calculus</b>	Maximum Marks : 30
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## (Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. at  $u = e^{xyz}$  then show that  $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)$  is it also equal to  $\frac{\partial^3 u}{\partial y \partial z \partial x}$  ?
2. Show that  $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$
3. A particle moves so that its position vector is given by  $\vec{r} = \hat{i} \cos wt + \hat{j} \sin wt$  Show that the velocity  $\vec{v}$  is perpendicular  $\vec{r}$  and  $\vec{r} \times \vec{v}$  is constant vector.

## (Section – B)

(Short Answer Questions)

Maximum Marks: 12

**Note :** Answer each question in 200 to 300 Words. All carry equal marks.

4. Find the deviational derivative of  $f(x) = xy^2 + yz^3$  at the point  $(1, -1, 1)$  along the vector  $\hat{i} + 2\hat{j} + 2\hat{k}$
5. at  $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
6. Determine the point where the function  $x^4 + y^4 - 2x^2 + 4xy - 2y^2$  has a maximum or minimum.
7. Find curl (curl  $\vec{F}$ ) at the point  $(0, 1, 2)$  where  $\vec{F} = (x^2 y)\hat{i} + (xyz)\hat{j} + (z^2 y)\hat{k}$   
Or  
Evaluate  $\int \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (3x^2)\hat{i} + (2xz - y)\hat{j} + z\hat{k}$  along the straight line joining  $(0, 0, 0)$  &  $(2, 1, 3)$