

LECTURE 22

MODE

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Mode is the most common item of a series. Generally, it is the value which occurs largest number of times in a series. In the words of **Croxton and Cowden**, "The mode of a distribution' is the value at the point around which the items tend to be most heavily concentrated."

According to **A.M. Tuttle**, 'Mode is the value which has the greatest frequency density in its immediate neighbourhood.'

The above two definitions indicate that mode is a value around which there is the greatest concentration of values. It may not necessarily be the value which occurs the largest number of times in a series, as in some cases, the point of maximum concentration may be around some other value. In some cases, there may be more than one point of concentration of values and the series may be bi-modal or multi-modal. We shall discuss these cases later.

The word Mode is derived from the French word (*la mode*) which means fashion or the most popular phenomenon. Mode, thus, is the most popular item of a series around which there is the highest frequency density. When we speak of the 'average student', 'average collar size', 'average size of a shoe', we are referring to mode. When we say that, on an average, a student spends Rs. 300 per month, we imply that a very large number of students spend around Rs. 300 per month. It is the value of mode. It is the most typical or fashionable value of the series.

Example . Find the mode of the following data relating to the weight of 10 students :

Sl. No.	Weight in pounds	Sl. No.	Weight in pounds
1	20	6	130
2	130	7	132
3	135	8	132
4	130	9	135
5	140	10	141

Solution

Weight in pounds	No. of Students
120	1
130	3
132	2
135	2
140	1
141	1
	10

Since item 130 occurs the largest number of times, it is the modal value.

If there are more than one point of concentration, mode cannot be found and the series is called bi-modal.

Grouping method : In discrete and continuous series, if the items concentrate at more than one value, attempts are made to find out the point of maximum concentration with the help of grouping method. In this method, values are first arranged in ascending order and the

frequencies against each value are written down. These frequencies are then added in two's and the totals are written in lines between the values added.

Frequencies can be added in two's in two ways :

- (i) By adding frequencies of items number 1 and 2; 3 and 4; 5 and 6 and so on.
- (ii) By adding frequencies of items number 2 and 3; 4 and 5; 6 and 7 and so on. After this, the frequencies are added in three's. This can be done in three ways:
 - (a) By adding frequencies of items number 1, 2 and 3, 4, 5 and 6, 7, 8 and 9 and so on.
 - (b) By adding frequencies of items number 2, 3 and 4, 5, 6 and 7, 8, 9, and 10 and so on.
 - (c) By adding the frequencies of items number 3, 4 and 5, 6, 7 and 8, 9, 10 and 11 and so on.

If necessary, frequencies can be added in four's and five's also. After this, the size of items containing the maximum frequencies are noted down and the item which has the maximum frequency the largest number of times is called the mode. If grouping has been done in case of continuous series we shall be in a position to determine the modal class by this process.

Example . Find the mode of the following series :

Size	Frequency	Size	Frequency
5	48	13	52
6	52	14	41
7	56	15	57
8	60	16	63
9	63	17	52

10	57	18	48
11	55	19	40
12	50	—	—

Solution

Location of mode by grouping

Size of Item (x)	Frequency (f)					
	(1)	(2)	(3)	(4)	(5)	(6)
5	48	100	108	156	168	179
6	52					
7	56	116		180		
8	60		123			
9	63	120		157	143	
10	57		105			
11	55	102		161		
12	50		93		143	
13	52	98		150		
14	41		120		172	
15	57	115		163		
16	63		100		140	
17	52	88		163		
18	48		40		163	
19	40					

The frequencies in column (1) are first added in two's in columns (2) and (3). Then they are added in three's in columns (4), (5) and (6). The maximum frequency in each column is indicated by thick letters. It will be observed that mode changes with the change in grouping.

Thus, according to column (I), mode should be 9 or 16. To find out the point of maximum concentration, the data can be arranged in the shape of table as follows:

Analysis Table

Columns	Size of item containing maximum frequency						
(1)			9				16
(2)			9	10		15	16
(3)		8	9				
(4)		8	9	10			
(5)			9	10	11		
(6)	7	8	9				
No. of times a size occurs	1	3	9	3	1	1	2

Since the size 9 occurs the largest number of times, it is the modal size or mode is 9.

Example . Find the mode from the following data :

Values	Frequency	Values	Frequency
Below 50	97	Below 30	60
Below 45	95	Below 25	30
Below 40	90	Below 20	12
Below 35	80	Below 15	4

Solution. The cumulative series would first be converted into a simple continuous series as follows :

Values	Frequency	Values	Frequency
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45—50	2	25—30	30 f_1
40—45	5	20—25	18 f_0
35—40	10	15—20	8
30—35	20 f_2	10—15	4

This series does not need grouping as modal class is very prominent. The maximum frequency 30 is against the class-interval 25—30 which is the modal class. Grouping would also give the same result. Hence,

$$\begin{aligned}
 Z &= l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} (l_2 - l_1) \\
 &= 25 + \frac{30 - 18}{60 - 18 - 20} (30 - 25) \\
 &= 25 + \left(\frac{12}{22} \times 5 \right) = 27.72
 \end{aligned}$$

Example 10. Modal marks for a group of 94 students are 54. Ten students got marks between 0—20, thirty students between 40—60 and fourteen students between 80—100. Find out the number of students getting marks between 20—40 and 60—80 if the maximum mark of the test were 100.

Solution.

Marks	No. of Students
0—20	10
20—40	x

40—60	30
60—80	y
80—100	14
	94

$$\text{Mode} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} (l_2 - l_1), \text{ Mode is given as } 54$$

$$\text{so } 54 = 40 + \frac{30 - x}{60 - x - y} (60 - 40)$$

$$\text{or } 14 = \frac{30 - x}{60 - x - y} \times 20 = \frac{600 - 20x}{60 - x - y}$$

$$\text{or } 840 - 14x - 14y = 600 - 20x \quad \text{or} \quad 6x - 14y = -240$$

The total number of students is 94. Therefore, the missing values $(x + y)$ would be $(94 - 10 - 30 - 14)$ or 40.

So, we have two equations :

$$6x - 14y = -240 \quad \text{or} \quad x + y = 40$$

If they are solved as simultaneous equation, we get :

$$6x - 14y = -240 \quad \dots \text{ (i)}$$

$$6x + 6y = 240 \quad \dots \text{ (ii)}$$

Subtracting equation (ii) from (i) we get :

$$-20y = -480 \quad \text{or} \quad y = 24$$

Since $x + y = 40$, therefore $x = 40 - 24$ or 16.

The missing values are, thus, 16 and 24.

Summary:

Mode:

Mode of a distribution is the value which has maximum frequency. Mathematically it is denoted as M_o

and defined as

$$M_o = l + h \left(\frac{f_m - f_1}{2f_m - f_1 - f_2} \right)$$

Where, the class which is having highest frequency is called the *modal class*, l is lower limit of modal class, f_m frequency of modal class, h is magnitude of modal class and f_1 frequency just preceding the modal class and f_2 frequency just succeeding the modal class.

Sometimes mode is also computed with the help of mean and median. For a symmetrical distribution mean, median and mode coincide and if the distribution is moderately asymmetrical, the mean, median and mode are approximately related by the formula:

$$\text{Mode} \cong 3 \text{ Median} - 2 \text{ Mean}$$

Relationship among Measures of Average

For symmetrically shaped distribution: Mean = Median = Mode

For positively-skewed distribution: Mean > Median > Mode

For negatively skewed distribution: Mean < Median < Mode