Uttar Pradesh Rajarshi Tandon
Open University

MFN -107

Biostatistics

NMDC-225

Uttar Pradesh Rajarshi Tandon
Open University

MFN -107

Biostatistics

BLOCK

1

RESEARCH METHODOLOGY AND STATISTICAL METHODS

Course Design Committee

Dr. (Prof.) Ashutosh Gupta — **Director**
School of Science, UPRTOU Prayagraj

Prof. Umesh Nath Tripathi — **Member**
Department of Chemistry
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. S.I. Rizvi — **Member**
Department of Biochemistry
University of Allahabad, Prayagraj

Prof. Dinesh Yadav — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. Sharad Kumar Mishra — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Dr. Ravindra Pratap Singh — **Member**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Dr. Dharmveer Singh — **Member/Secretary**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Course Preparation Committee

Dr. Shruti — **Writer**
Sr. Assistant Professor, School of Sciences
U. P. Rajarshi Tandon Open University, Prayagraj

Prof. G. S. Pandey — **Editor**
Department of Statistics, University of Allahabad,
Prayagraj

Dr. Dharmveer Singh — **Course Coordinator**
Academic Consultant (Biochemistry)
School of Science, UPRTOU, Prayagraj

© **UPRTOU, Prayagraj - 2024**
ISBN - 978-93-94487-67-3

Printed and Published by Vinay Kumar, Registrar, Uttar Pradesh Rajarshi Tandon Open University, Prayagraj - 2024.

Printed by - K. C. Printing & Allied Works, Panchwati, Mathura -281003

BLOCK INTRODUCTION

The present SLM on ***Bio Statistics*** consists of three Blocks. *Block - 1 – Research Methodology and Statistical Methods* has two units; *Block - 2 – Probability - Distributions and Demography* has two units; and at the last *Block – 3 Tests of Significance and Analysis of Variance* has two units.

The ***Block-1– Research Methodology and Statistical Methods*** consists two units. The *first unit* of this block; named *Research Methods and Sampling Procedures*; describes the meaning and types of research, significance of research. It tells about the research problem and its selection with *Sampling Theory*; which discuss about the sampling, different types of sampling designs, simple random sampling, stratified sampling and cluster sampling with their applications.. The *second unit* of this block; named *Statistical Tools*; discusses about the measures of central tendency, measures of dispersion, measures of asymmetry, correlation and regression analysis, association of attributes and 3-sigma limits.

The ***Block-2– Probability- Distributions Theory and Demography*** is the second block having two units. The *first unit* of this block; named *Probability and Distribution Theory*; gives the Basic concepts of probability, definitions of probability, additive and multiplicative law of probability, conditional probability, Bayes' theorem, random variable and its types, probability mass function and probability density functions. In *Probability Distributions*, the concept of probability distribution, discrete and continuous probability distributions namely Binomial Distribution, Poisson Distribution, Geometric Distribution, Normal Distribution, Exponential Distribution have been also discussed in this unit along with their properties, applications and importance.. The *second unit* of this block; named *Demography*; gives knowledge about the vital statistics and demography, this also tells about the source of vital statistics and demographic data, rates, ratio, proportion, measures of fertility, measures of mortality, measures of morbidity and migration.

The ***Block-3– Tests of Significance and Analysis of Variance*** consists of two units. The *first unit* of this block; named *Testing of Hypothesis*; discuss about the hypothesis and its types, level of significance, critical region, p-value, types of errors, chi-square tests, t-tests and z-tests with their applications. The *second unit* of this block; named *Analysis of Variance*; discusses about the concept of analysis of variance and co-variance, basic principles of ANOVA and ANCOVA. (One Way, Two Way and Three Way Analysis).

Illustrations and examples on these topics have also been given.

At the end of every block/unit the summary, self assessment questions and further readings are given.

Unit-1 Research Methods and Sampling Procedures

Structure

1.1 INTRODUCTION

Now a day's research plays an important role in higher studies. Several research studies are published every year. But some are cached more attention and some are less. A research which follows systematic and scientific procedures, gain more attention and found better results. This unit blows some light on all these.

1.2 OBJECTIVES

After studying this unit you will be able to

- Understand the meaning and types of research.
- Understand the significance of research, research problem and its selection.
- Understand the sampling procedures and sampling techniques.

1.3 MEANING AND TYPES OF RESEARCH AND RESEARCH DESIGN

Research is a method of seeking the new information, new thoughts, and new knowledge. Research is regularly used for solving problems and increasing the knowledge. This can be completed by theories and observations. Research is to be systematic, organized, objective and scientific.

The two basic research ***types*** are quantitative and qualitative research. But most of the time research can be separated into following different categories;

- Exploratory
- descriptive
- analytical
- causal
- Applied
- Fundamental
- Conceptual
- Empirical, etc

A ***research design*** is a overall strategic, planed framework for concluding the research scientifically through the collection, interpretation, analysis, and discussion of data. There are some major types of research design are as follows:

- General Structure and Writing Style
- Action Research Design
- Case Study Design
- Causal Design
- Cohort Design
- Cross-Sectional Design
- Descriptive Design
- Sampling Design
- Statistical Design
- Experimental Design

1.4 OBJECTIVE AND CHARACTERISTICS OF A GOOD RESEARCH

The ***objective*** of a research is to discover answer to question through the application of scientific and statistical procedures. The main aim of any research is to find out something new, which is hidden and also not been discovered as up till now.

A good research design should always accomplish the flowing main ***characteristics***:

- It should be systematically, analytical, critical, methodical, and reflexive.
- It has a systematic statistically process of collecting and analyzing the data.
- It is methodological, empirical and replicable.
- It should be reliable.

1.5 RESEARCH PROBLEM

A ***research problem*** is a condition to be improved, a difficulty to be eliminated, or a troubling question that exists in scholarly literature, in theory, or in practical those points to the need for important concerned and focused study/research. In simple words we can say, a research problem refers to the difficulty which a researcher experiences in his theoretical or practical study. It has been seen that, the research problems are of three types: descriptive, relational, and causal/fundamental research. Thus a research problem helps researcher to invent the systematic chain of study. It also helps to avoid unnecessary steps during the research period. It is the beginning step in research study. It helps to understand the research procedure in a better scientific and systematic manner.

1.6 SAMPLING PROCEDURES

Before going to study of sampling theory, blow some lights on its terminology. It is known that *population* is a 'well explained groups which is being studied' and a *sample* is the "Small collection of the population" which has actually been observed. Before studying the theory of sampling, it is necessary to know about the *parameter* and *statistic.* The measures (like as mean, variance, correlation coefficient, etc.) of population are known *parameters* of the population. Same as these measures which drawn for sample is known as *statistic*. Therefore parameter and statistic are two most important elements of further study. It is known that a part of the population which is representative of the population is known as *sample* and the process of selection is known as *sampling* and also the relation between sample and population, with all statistical principles and theories are known as *sampling theory.* If population is very large, then it is single useful method which can be applicable. There are following types of sampling:

1.7 SAMPLING TECHNIQUES

Random and Non Random Sampling- A sampling procedure is said to be *random,* when it follow the scientific methods for selecting the samples. In this each unit of the population has an equal chance of selection in the sample.

A sampling procedure is said to be *non-random* when it do not chase any

scientific methods and has not an equal chance of selection. In other words the sample units are selected without use of randomization.

Note:

- Some sampling procedures of random Sampling are Simple Random Sampling, Stratified Sampling, Systematic Sampling, Cluster Sampling, and Multistage Sampling etc..
- Some sampling procedures of Non-random Sampling are Judgmental Sampling, Convenience Sampling, Quota Sampling, and Snowball Sampling etc..
- In the study of sampling theory, there are some possibilities to the occurrence of two types of errors, say *sampling errors* and *non sampling errors*.

In this unit we study only flowing random sampling procedures.

1.7.1 SIMPLE RANDOM SAMPLING (SRS)

A sampling in which, every unit of the population (here population must be finite and homogeneous) has equal independent chance of selection in sample. There are two types of simple random sampling, when

- The sample units are selected without replacement (no element can be selected more than once in the same sample), is known as *Simple random sampling without replacement (SRSWOR).*
- The sample units are selected with replacement (an element may appear multiple times in the same sample), is known as *Simple random sampling with replacement (SRSWR).* (Practically it is not used for further analysis).

There are some methods for selecting the SRS:

Lottery Method- This is most popular and the simplest method of selection of sample from population. In this method all objects of population are numbered on the slips (which are same in shape, size and colour). These slips are shuffled well and a blindfold selection is made.

Random Number Table Method- If the population size is very large then above method is impossible. Therefore random numbers table may be used. Some well known random number tables are, Tippet's random number table, Fisher and Yates table, Kendall and Smith's table, etc.

1.7.2 STRATIFIED SAMPLING

This method is useful when the nature of population is heterogeneous (i.e. not homogeneous). A sample which is drawn after stratification follows two steps mainly; at the first it is to divide the entire heterogeneous population into several non-overlapping homogeneous sub-populations/ groups, these homogeneous groups or classes known as *strata* and secondly from each *stratum*, the units are drawn by SRS, and lastly combining all units collect together. This technique

gives more representative sample than SRS in a large heterogeneous population. In other words, it gives a proportionate representative sample from each group is secured and it gives greater accuracy.

1.7.3 CLUSTER SAMPLING

In this sampling procedure, the whole population is divided into sub population (separate groups of units) say *clusters,* where each unit of the population belongs to one and only one cluster. A SRS is taken from each cluster. Whenever the elements within cluster are heterogeneous, it tends to provide best results. In this each cluster is the representative of the entire population. It is also called ***Area Sampling*.**

1.8 SUMMARY

This unit gives detailed knowledge about the meaning and types of research, significance of research. It tells about the research problem and sampling theory discuss about the sampling, different types of sampling designs, simple random sampling, stratified sampling and cluster sampling with their applications.

1.9 CHECK YOUR PROGRESS

1. Discuss about the meaning and types of research?
2. Discuss about the research problem?
3. What do you mean by sampling procedures and its techniques?
4. Write a short note on:
   a. Simple Random Sampling
   b. Stratified Sampling
   c. Cluster Sampling

1.9 FURTHER READINGS

1. Goon, Gupta & Dasgupta: Fundamentals of Statistics Vol. II the World Press Pvt. Ltd., Kolkata.
2. Kothari, C. R., Research Methodology: Methods and Techniques; New Age International Publishers, New Delhi.
3. Kothari, C. R., Quantitative Techniques; Vikas Publishing House, New Delhi.
4. Gupta S. C., Kapoor, V.K., Applied Statistics, S. Chand, New Delhi.

UNIT-2 STATISTICAL METHODS

Structure

2.1 INTRODUCTION

Statistical Methods is a wide-ranging term which includes almost all the methods involves in the collection, processing condensing and analyzing of data. The data collected from the field for a number of items vary greatly in their qualitative as well as quantitative nature. In this unit have been highlighted different measures of central tendency, measures of dispersion, measures of asymmetry, measures of relationships are covered.

2.2 OBJECTIVES

After studying this unit you will be able to

- Understand the meaning of central tendency of data and compute common measures of central tendency.
- Understand the meaning of dispersion of data and compute common measures of disperson.
- Understand the meaning of symmetry of data and compute common measures of asymmetry.
- Understand the meaning of relationships of data and compute common measures of relationship.

2.3 MEASURES OF CENTRAL TENDENCY AND ITS TYPES

When the observations have a trend to cluster round a central value, and this feature of observations is called the "**Central Tendency"** and related statistical measure (which gives the point round which the observation has a tendency to cluster) is known as '**measure of central tendency'.** The central value of the variable in any series of observations is useful in finding the location of the distribution and so it is also called an **average**.

Seeing as each one of the given measures of central tendency has its own individual characteristics and properties, a decision must always be make as to which would be the most appropriate and useful in view of the nature of the statistical data and purpose of the inquiry. The qualities preferred in a measure should be rigidly defined, easily computed, capable of a simple interpretation, not influenced by one or two extremely large or small values and likely to fluctuate relatively little from one random sample to another (of the same size and from the same population).

The measures of central tendency or averages are of different types, but the most common in use are of three types:

1. Mean
2. Median
3. Mode

The mean is further classified as:

(i) Arithmetic mean

(ii) Geometric mean

(iii) Harmonic mean

2.3.1 ARITHMETIC MEAN (FOR UNGROUPED DATA)

The arithmetic mean of a series of n observations x_1, x_2, x_3, ……x_n is obtained by summing up the values of all the observations and dividing the total by the number of observations. Thus,

$$X \quad \frac{Sum\ of\ observations\ \ or\ values}{Number\ of\ obsevations}$$

$$\frac{x \quad x \quad x \ \ldots\ldots \quad x}{n} \quad \frac{\sum \quad x}{n}$$

Where $\sum$ (sigma) stands for summations and x_i is the i^{th} value of the observation (variable).

Example

The rainfall record in a month of 10 regions of a State is given below. Compute the average rainfall of the month for the State.

Region	1	2	3	4	5	6	7	8	9	10
Rainfall	17.6	10.1	11.4	18.5	10.5	14.3	8.9	13.4	10.6	12.5

(in mm)

Solution :

$$Mean \quad X \quad \frac{\sum \quad x}{10} \quad \frac{x \quad x \quad x \ \ldots\ldots \quad x}{10}$$

$$\frac{17.6 \quad 10.1 \quad 11.4 \quad \cdots .10.6 \quad 12.5}{10} \quad \frac{127.8}{10} \quad 12.78mm$$

The computation of Arithmetic mean using short cut method is discussed below:

Short-Cut Method- This method is applied to avoid lengthy calculations.

Let $x \quad x \quad x \ \ldots\ldots \quad x$ be n individual reading on the variable and let A be the working mean. Let d_1, d_2, d_3…..d_n denote the differences between the working mean and individual values $x \quad x \quad x \ \ldots\ldots \quad x$ respectively. The mean $\bar{x}$ of X, in terms of the mean $\bar{d}$ of differences, is calculated as:

$$d \quad \frac{d \quad d \quad d \ \ldots\ldots \quad d}{n}$$

$$\frac{x \quad A \quad x \quad A \quad x \quad A \quad \cdots\ldots \quad x \quad A}{n}$$

$$\frac{\sum \quad x \quad nA}{n} \quad \frac{\sum \quad X}{n} \quad A \quad X \quad A$$

Or, $X \quad A \quad \bar{d} \quad A \quad -\sum \quad d$

An illustration is given below.

Example: Calculation the mean for the following scores: 60, 65, 74, 85, 95.

Solution:

Table

x_i (Scores)	x_i-74
60	-14
65	-9
74	0
85	+11
95	+21
	+9

Then, mean score is

$$\bar{x} \quad 74 \quad 9/5 \quad 74 \quad 1.8 \quad 75.8$$

Grouped data (Discrete Frequency Distribution)- In a discrete series, let the individual readings $x \ , \ x \ , \ x \ \ldots\ldots x$ of the variable X occur (have frequencies) $f \ , \ f \ , \ f \ \ldots\ldots, f$ times respectively. Then the mean of X is obtained by summing the product of individual readings with corresponding frequencies and dividing the total by the sum of frequencies, i.e.,

$$X \quad \frac{x \ f \quad x \ f \quad x \ f \ \ldots\ldots \quad x \ f}{f \quad f \quad f \ \ldots\ldots \quad f} \quad \frac{\sum \quad X \ f}{\sum \quad f} \quad \frac{\sum \quad X \ f}{N}$$

Where N= $\sum \quad f$ is the total number of observations.

Short cut method- The arithmetic mean ($\bar{x}$) is then calculated with the help of following formula:

$$Mean \quad \bar{x} \quad A \ \frac{1}{n} \quad f \ d$$

Example- Below are given the number of children born per family in 735 families in a locality. Calculate the average number of children born per family in the locality.

Table

Number of children born per family	Number of families
0	96
1	108
2	154
3	126
4	95
5	62
6	45
7	20
8	11
9	6
10	5
11	5
12	1
13	1

Solution- Computation of the average number of children born per family:

Table

Number of children born per family (X)	Number of Families (f)	$X.f$ xf
0	96	0 96=0
1	108	1 108=108
2	154	2 154=308
3	126	3 126=378
4	95	4 95=380
5	62	5 62=310
6	45	6 45=270
7	20	7 20=140
8	11	8 11=88
9	6	9 6=54
10	5	10 5=50
11	5	11 5=55
12	1	12 1=12
13	1	13 1=13
Total	735	2166

Here N = $\sum f$= 735; $\sum Xf$ =2166

Average number of children born per family is given by

$$mean \quad x \qquad \frac{\sum xf}{\sum f} \quad \frac{2166}{735} \qquad 2.9 \; children$$

Grouped Data (Continuous Frequency Distribution)- When the measurement are given in the grouped form, the mean is computed by multiplying the various mid values m_i with their respective frequencies f_i where i= 1,2,…..n and dividing the product total by the sum of frequencies or total number of observations. If m_1, m_2, ……. m_n are the mid values or class intervals corresponding to frequencies f_1, f_2,….f_n then the mean is

$$X \quad \frac{m \quad m\,f \quad \cdots\ldots \quad m\,f}{f \quad f \ldots\ldots \quad f} \quad \frac{\sum \quad m\,f}{\sum \quad f} \quad \frac{\sum \quad m\,f}{N}$$

Where, n stand for the number of groups and N denotes the total number of observations.

Short Cut Method- If the class groups formed by individual reading are large, calculate arithmetic mean $\bar{x}$ by the following formula:

$$mean \quad \bar{x} \quad A \quad \frac{\sum f d}{N}$$

Step- Deviation Method- if d ———, the formula for calculating arithmetic mean by this method is

$$mean \quad \bar{x} \quad A \quad \frac{\sum f d}{N} \quad h$$

Where h is the size of the class intervals.

Example- Calculate mean from given table

Height (in inches)	0-10	10-20	20-30	30-40	40-50	50-60
No. of plants	22	10	8	15	5	6

Solution-

(a) By direct method

Height (in inches)	*Mid Values (m)*	No. of plants	*m.f (mf)*
0-10	$\frac{0 \quad 10}{2}$ 5	22	5 22 110
10-20	$\frac{10 \quad 20}{2}$ 15	10	15 10 150
20-30	$\frac{20 \quad 30}{2}$ 25	8	25 8 200
30-40	$\frac{30 \quad 40}{2}$ 35	15	35 15 525
40-50	$\frac{40 \quad 50}{2}$ 45	5	45 5 225
50-60	$\frac{50 \quad 60}{2}$ 55	6	55 6 330
Total		66	1540

$$Here\ N \quad mf \quad 1540$$

$$mean\ \bar{x} \quad \frac{\sum mf}{\sum f} \quad \frac{1540}{66} \quad 23.33\ inches$$

(b) Short cut method

Height (in inches)	*Mid Values (m)*	*d=m-A*	No. of plants	*d.f (df)*
0-10	5	-30	22	30 22 660
10-20	15	-20	10	20 10 220
20-30	25	-10	8	10 8 80
30-40	35(A)	0	15	0 15 0
40-50	45	10	5	10 5 50
50-60	55	20	6	20 6 120
Total			66	-770

$$Here\ N \quad f \quad 66, \quad df \quad 770$$

$$mean\ X \quad A \quad \frac{\sum fd}{N}$$

$$35 \quad \frac{770}{66} \quad 23.33\ inches$$

(c) By step- deviation Method

From the given data *h*=10

Height (in inches)	*Mid Values (m)*	$d \quad \frac{m \quad A}{h}$	No. of plants	*d.f (df)*
0-10	5	$\frac{5 \quad 35}{10}$ 3	22	3 22 66
10-20	15	$\frac{15 \quad 35}{10}$ 2	10	2 10 20

20-30	25	$\frac{25 \quad 35}{10}$ 1	8	1 8 8
30-40	35(A)	$\frac{35 \quad 35}{10}$ 0	15	0 15 0
40-50	45	$\frac{45 \quad 35}{10}$ 1	5	1 5 5
50-60	55	$\frac{55 \quad 35}{10}$ 2	6	2 6 12
Total	-	66	66	-77

$$Here\ N \quad f \quad 66, \quad fd' \quad 77$$

$$mean\ \ X \quad A \ \frac{\sum fd'}{N} \quad h \quad 35 \ \frac{77}{66} \quad 10 \quad 23.33\ inches$$

Properties of Arithmetic Mean-

1. ***First Property of Mean-*** *The sum of the deviations about the arithmetic mean equals zero.*

Mathematically

$$f \ x \quad \bar{x} \quad 0$$

This property says that if the mean is subtracted from each score, the sum of the differences will equal zero. The property results from the fact that the mean is the balance point of the distribution. The mean can be thought of as the fulcrum of a seesaw. When the score are distribution along the seesaw according to their values, the mean of the distribution occupies the position where the scores are in balance. This is known as first property of mean.

2. ***Second Property of Mean-*** *The sum to the squared deviations of all the scores about their arithmetic mean is minimum.*

That is,

$$f \ x \quad \bar{x} \quad minimum$$

This is an important characteristic and is used in many areas of statistics, particularly in regression analysis.

3. ***Combined (Additive) Property of Mean-*** *If X and X be the means of two series of sizes n_1 and n_2 respectively, then the mean of the combined series can be computed as:*

$$X \quad \frac{n\,X \quad n\,X}{n \quad n}$$

In the same way, if $X\ ,X\ ,X\ \ldots\ldots X$ be the means of k series of sizes $n\ ,n\ ,n\ \ldots\ldots n$ respectively then the mean X of combined series is

$$X \quad \frac{n\,X \quad n\,X \quad \cdots\ldots\ldots n\,X}{n \quad n \quad \ldots\ldots n}$$

Example- The average ages of 250 males and 210 females in a village are 41.6 and 38.5 years respectively. Find the average age combining both males and females together.

Solution- Here (Combined average) is N_1= 250, X = 41.6 years and N_2= 210, X = 38.5 years. Therefore,

$$X \quad \frac{n\,X \quad n\,X}{n \quad n} \qquad \frac{250 \quad 41.6 \quad 210 \quad 38.5}{250 \quad 210} \qquad \frac{10400 \quad 8085}{460}$$

$$40.18\ years$$

2.3.2 GEOMETRIC MEAN

In case of finding the average or rates and ratios, geometric mean is more useful measure than others, e.g. in finding the population increase simple and compound interests etc.

Case 1- In case of ungrouped data it is obtained by multiplying together all the values of the variable and extracting the relevant root of the product. i.e. if $x\ ,x\ ,x\ \ldots.x$ and n values of a variable under study, then the geometric mean (G.M.) is computed as:

$$\text{G.M.} \quad x\ ,x\ ,x\ \ldots.x\ ;\quad x \quad 0$$

To facilitate the computation one can make the use of logarithms as:

$$\log\ G.M. \quad \frac{1}{n}\log\ x\ ,x\ ,x\ \ldots.x$$

$$\frac{1}{n}\ log x \quad log x \quad log x\ \ldots..\ log x \quad \frac{1}{n} \quad log x$$

$$\text{So G.M.} \quad \text{Antilog}\ \frac{1}{n} \quad log x$$

Thus the logarithm of the geometric mean is equal to the arithmetic mean of the logarithms of individuals' measurements.

Case 2- In case of grouped data if $f\ ,f\ ,f\ \ldots.f$ be the frequencies corresponding to the individual values $x\ ,x\ ,x\ \ldots.x$ then G.M. is computed as:

$$G.M. \quad x.x \ .x \ \ldots..x^{\Sigma /} \quad ; \quad x \quad 0$$

$$or \log G.M. \quad \frac{1}{\Sigma \quad f} \ f \ log\mathrm{x} \quad \cdots.. \quad f \ log\mathrm{x} \quad \frac{1}{\Sigma \quad f} \quad f \ \ logx$$

$$So \ G.M. \quad Antilog \ \frac{1}{\Sigma \quad f} \quad f \ \ logx$$

Here log G is the weighted mean of log $_G$ y_i, s with weights $f_1, f_2, \ldots..f_n$.

Additive Property of Geometric Mean- If G_1 and G_2 are the geometric means of two diffrent series with respective sizes n_1 and n_2, the combined Geometric mean G is

$$logG \quad \frac{n \ logG \quad n \ logG}{n \quad n}$$

Example- The monthly average temperature of a situation for five months is given as 16.2, 23.4, 20.6, 33.4 and 16.4 degree centigrade. Find the mean temperature of the station.

Solution- $G.M.$ 16.2 23.4 20.6 33.4 16.4 /

or

$$\log G.M. \quad \frac{1}{5} \log 16.2 \quad \cdots \quad \log 16.4$$

$$\frac{1}{5} \ 1.2095 \quad 1.3692 \quad 1.3139 \quad 1.5238 \quad 1.2148$$

$$\frac{1}{5} \quad 6.6312 \quad 1.3262$$

so G.M. = Antilog [1.3262]= 21.1934 degree centigrade.

Example- From the following data, calculate the G.M.

Height (in inches)	0-10	10-20	20-30	30-40	40-50
No. of plants	14	23	27	21	15

Calculation-

Height (in inches)	*Mid Values (x)*	*log (x)*	*No. of observations (f)*	*f log x*
0-10	5	0.69897	14	9.78558
10-20	15	1.1769	23	27.05007

20-30	25	1.39794	27	37.74438
30-40	35	1.54407	21	32.42547
40-50	45	1.65321	15	24.79815
Total	-		100	131.80365

Here $\sum f$=N= 100, $\sum f \log x$ = 131.80365

$$G.M. \quad Antilog \quad \frac{1}{\sum f} \quad f\ logx$$

$$Antilog \quad \frac{131.80365}{100} \qquad Antilog 1.318036 \qquad 20.7987$$

2.3.3 HARMONIC MEAN

In problems such as work time and rate where the amount of work is held constant an average rate is required, the harmonic mean (HM) is utilized. It is defined as the reciprocal of the arithmetic mean of the reciprocals of the given individual readings i.e. H.M. of $x, x \ldots x$ is defined as:

$$H.M. \quad \frac{n}{\frac{1}{x} \quad \frac{1}{x} \quad \cdots \frac{1}{x}} \qquad \frac{n}{\sum \quad \frac{1}{x}} \qquad\qquad x \quad 0$$

Where n is the number of observations.

Example- The H.M. of 2, 4, 6 is

$$H.M. \quad \frac{3}{\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{6}} \quad \frac{36}{11} \quad 3.37$$

In case of frequency distribution (grouped data), if $f, f \ldots f$ be the frequencies corresponding to $x, x \ldots x$ then H.M. is computed as:

$$H.M. \quad \frac{f \quad f \quad f \quad \cdots \quad f}{\frac{f}{x} \quad \frac{f}{x} \quad \cdots \quad \frac{f}{x}} \qquad \frac{\sum \quad f}{\sum \quad \frac{f}{x}} \qquad\qquad x \quad 0$$

Example Calculation-

Class group (X)	*Mid Values (m)*	*No. of observations (f)*	*f / m*
0-10	5	14	2.80
10-20	15	23	1.53

20-30	25	27	1.08
30-40	35	21	0.60
40-50	45	15	0.33
Total		100	6.34

Here $\sum f$=N= 100, $\sum f / m$ = 6.34

$$H.M. \quad \frac{\sum f}{\sum \frac{f}{m}} \quad \frac{100}{6.34} \quad 15.77$$

2.3.4 MEDIAN

Median is another important and useful measure of central tendency. It has connotation of the middle most or most central value of a set of measurements. It is usually defined as the value which divides a distribution in such a manner that the number of items below it is equal to the number of items above it. The median is thus a positional average. Median is that variate value of the data or frequency distribution which divides it in two equal parts.

Calculation of Median (Ungrouped data)- ***Case 1 (n is odd):*** In case of ungrouped data when the number of observations are odd, then median is the middle value after the measurements have been arranged in ascending or descending order of magnitude, i.e. if there are n number of measurements and measurements are arranged in ascending or descending order of magnitude, the median of the measurements is —— measurement where n is an odd number.

Case 2 (n is even)- If the numbers of observations are even, median is defined as the mean of the two middle observations, when observations are arranged in ascending or descending order to magnitude i.e.

$$median \quad \frac{\frac{n}{2} \quad value \quad \frac{n \quad 1}{2} \quad value}{2}$$

Example- Calculate median for the following data:

(a) 68, 62, 75, 82, 68, 71, 68, 71, 62, 68, 74, 59, 74, 68, 60, 71, 59, 73, 73, 58.

(b) 200, 150, 260, 285, 380, 305, 4989, 307, 1280, 233, 403

Solution-

(a) To compute the median first we arrange the values in ascending order of magnitude as:

58, 59, 59, 60, 62, 62, 68, 68, 68, 68, 68, 71, 71, 71, 73, 73, 74, 74, 75, 82.

The number of observation n is even in this case, i.e., n=20

So

$$median \quad \frac{\frac{n}{2} \quad value \quad \frac{n \quad 1}{2} \quad value}{2} \quad \frac{10th\ value \quad 11th\ value}{2}$$

$$\frac{68 \quad 68}{2} \quad 68$$

(b) Let us first arrange the values in ascending order of magnitude as:

150, 200, 233, 260, 285, 305, 307, 380, 403, 1280, 4989

The number of observations n in this case is odd, i.e., n=11 so the median is the —— value i.e., —— or 6^{th} value of the observation and thus underlined value. i.e., 305 is the median.

Calculation of Median (Grouped Data)- In case of discrete frequency distribution median can be obtained with the help of cumulative frequencies as follows:

(i) First find N/2 where N=$\sum f$

(ii) Find the cumulative frequency just greater then N/2.

(iii) Corresponding value of X (i.e., of variable) is median.

Example Calculation-

Height (in metre) (X)	Number of units (f)	Cumulative frequency f
200	142	142
600	265	407
1000	560	967
1400	271	1238
1800	89	1327
2200	16	1343
Total	1343	

Here f=N= 1343; – 671.5

The cumulative frequency just greater than 671.5 is 967 and corresponding to this cumulative frequency, the value of X is 1000 and thus the median height is 1000 meters.

Median (Continuous Grouped Data)- Median for such distribution is computed by the following formula

$$Median\ \ Md \quad l \quad \frac{\frac{n}{2} \quad f}{f}.h$$

Where l_m is the lower limit f_m is the frequency of the median class, f_e is the cumulative frequency of the class, preceding the median class and h is the class width of the median class and N =$\sum f$

Example- Calculate median for the following grouped data.

Interval	35-45	45-55	55-65	65-75	75-85
Frequency	2	3	5	1	1
Cum. Freq.	2	5	10	11	12

The cum. Freq. is computed from the given freq. dist.

The median position = (n+1)/2= (12+1)/2 = 6.5

Median lies between observation 55 and 65. Both of these observations fall in category 3, i.e., in class (35-65) with cumulative frequency of 10. Therefore,

$$Median\ \ Md \quad l \quad \frac{\frac{n}{2} \quad f}{f}.h$$

Where l_m =55, f_m =5, h=10, f_e =5, N=12

$$Median\ \ Md \quad 55 \quad \frac{12/2 \quad 5}{5} \quad 10 \quad 55 \quad 2 \quad 57$$

2.3.5 MODE

Mode or modal value of a distribution is that the value which occurs most frequency. In case of frequency distribution the mode is that value which has maximum frequency. If two or more observations occur the same number of times then there is more than one mode and the distribution is called multi-model as against uni-model.

Calculation of Mode (Ungrouped Data)- Mode is defined as that variate value of the data or the frequency distribution which occurs most frequently.

Example- Find the modal temperature value from the values given

Solution-

(i) Putting data in array as:

58, 59, 59, 60, 62, 62, 68, 68, 68, 68, 68, 71, 71, 71, 73, 73, 74, 74, 75, 82.

Here mode = 68^0 F.

(ii) Discrete series (converted to frequency distribution form)

Variable (X)	58	59	60	62	68	71	73	74	75	82
Frequency (f)	1	2	1	2	5	3	2	2	1	1

Here the value 68 occurs the maximum number of times, hence it is mode.

Discrete Series (Grouped Data)- In case of discrete frequency distribution, mode can be located by inspection of the distribution alone. The size having the maximum frequency will be reckoned as mode.

Continuous Series (Grouped)- Determine the value of mode by applying the following formula:

$$Mode \quad L \quad \frac{f \quad f}{2f \quad f \quad f} \quad h$$

Where, L is the lower limit of the modal class; f is the frequency of the modal class; f_P is the frequency of the class preceding the modal class ;f_s is the frequency of the class succeeding the modal class and h is the class width of the modal class.

Example- Compute the modal agricultural holding of the village from the data given in above example

$$Mode \quad L \quad \frac{f \quad f}{2f \quad f \quad f} \quad h$$

if $L= 14.5, f= 150, f_P= 35, f_S= 70$ *and* $h=5$.

$$Mode \quad 14.5 \quad \frac{150 \quad 35}{150 \quad 2 \quad 35 \quad 70} \quad 5$$

$$14.5 \quad \frac{115}{195} \quad 5 \quad 17.45 \; acres.$$

Relationship between mean, Median and Mode- For a symmetrical distribution mean, median and mode coincide and if the distribution is moderately asymmetrical, the mean, median and mode are approximately related by the formula:

Mode ≅ 3 Median - 2 Mean

2.4 MEASURES OF DISPERSION

The measures of dispersion summarize the variability of the distribution. It is useful to know how similar of dissimilar scores are from the average score and form one another. We might like to know if scores cluster, they are more homogeneous. If scores are spread out widely then they are more heterogeneous. The measures of central tendency as discussed above only locate the center of a distribution but tell nothing about the degree of variability. In order to study the dispersion or variability in a distribution, we need alternative measures called the measures of dispersion. In central tendency is thought of as the point that best represents a central score in a distribution, the dispersion presents the other side of the coin.

Types of Measures of Dispersion- Dispersion is defined as the degree to which scores deviate from the central tendency (usually the mean) of the distribution. The statistical techniques that quantify this dispersion in a distribution are called measures of dispersion. Most commonly used measures of dispersion are range, average deviations, variance and standard deviation.

2.4.1 RANGE

Range is defined as the difference between the highest and the lowest scores in a distribution. Symbolically,

$R = X_{max} - X_{min}$

Where R is the range, X_{max} is the highest score, X_{min} is the lowest score.

Example- Find range for value: 87, 92, 47, 58, 87, 62, 73, 73, 61.

Solution- It is always a good idea to first rank the observation in ascending or descending order. In an ascending order the scores are: 47, 58, 61, 62, 73, 73, 87, 87, 92. A visual examination shows that $X_{max}= 92$; $X_{min}= 47$.

Therefore R=92-47=45

2.4.2 MEAN DEVIATION

The formula for average deviation can be written as:

$$MD \qquad |x \quad \bar{x}| \quad /n$$

Where $\bar{x}$ is the arithmetic mean; $x \quad \bar{x}$ is deviation of from $\bar{x}$ and $|x \quad \bar{x}|$ is the absolute value of the deviation which is always a positive number.

Example- The numbers of terms that five randomly selected Members of Parliament have served are: 3, 10, 12, 7, 8. Find the average deviation of these scores.

Solution- Make the following table containing the calculation.

Case number	Terms	$x \quad \bar{x}$	$\lvert x \quad \bar{x} \rvert$
1	3	3-8=-5	5
2	10	10-8=2	2
3	12	12-8=4	4
4	7	7-8=-1	1
5	8	8-8=0	0
Total	40	0	12

Mean $\bar{x}$ = 40/5=8

MD= 12/5=2.4 terms.

2.4.3 VARIANCE AND STANDARD DEVIATION

Variance may be defined as the mean squared deviation of scores around the mean. In the form of a formula, variance is given by:

$$S \qquad \frac{x \quad \bar{x}}{n} \qquad \text{for sample data}$$

$$\sigma \qquad \frac{x \quad \mu}{n} \qquad \text{for population data}$$

Where S = sample variance of variable X; σ is population variance x_i is the value of X variable for i^{th} case; $\bar{x}$ is sample mean; μ is population mean; n is the sample size; and N is population size.

Standard Deviation- The square root of the variance is called the standard deviation (SD), represented by s or σ. That is Standard deviation = Square root of variance or s= $\sqrt{s}$

$$s \qquad \overline{s} \qquad \overline{\frac{x \quad \bar{x}}{n \quad 1}} \qquad \overline{f \ \frac{x \quad \bar{x}}{n \quad 1}}$$

Similarly, population standard deviation $\sigma \quad \sqrt{\sigma}$

Effect of Change or Origin and Scale- This S.D. is

$$S \qquad \overline{\frac{x \quad \bar{x}}{n \quad 1}}$$

Where $\bar{x} \quad -\sum x$

If δ —— $\forall i \quad 12, \ldots \ldots n,$

$S \qquad hS$

i.e., standard deviation is independent of change of origin but not independent of change of scale.

Example- Calculate the mean and S.D. for the following given table of marks distribution of 50 students

Marks	Students (f)	Mid value (x)	x_i-25	δ =——	$f\,\delta$	$f\,\delta$
0-10	2	5	5-25=-20	-2	-4	8
10-20	10	15	15-25= -10	-1	-10	10
20-30	15	25	25-25=0	0	0	0
30-40	14	35	35-25=10	1	14	14
40-50	9	45	45-25=20	2	18	36
	50				18	68

$$\bar{x}\ \delta \quad a \quad \frac{\sum f\,\delta}{N} \quad N \quad f$$

$$25 \quad \frac{10 \quad 18}{50} \quad 25 \quad 3.6 \quad 28.6$$

$$S \quad hS$$

$$S \quad \frac{1}{N} \quad f \quad \delta \quad \bar{x} \quad \frac{1}{N} \quad f \quad \delta \quad \frac{1}{N} \quad f \quad \delta$$

$$\frac{1}{50} \quad 68 \quad \frac{1}{50} \quad 18 \quad 1.36 \quad 0.36 \quad 1.36 \quad 0.1296 \quad 1.4896$$

$$S \quad 10 \quad 1.49 \quad 14.896$$

Combined Variance Property- If n_1 and n_2 be the sizes of two series with respective means X , X and respective variances then the standard deviations S , S of the combined series is denoted as S and defined as

$$\boldsymbol{S^2 \quad \frac{1}{n_1 \quad n_2} \quad n_1 \quad S_1^2 \quad d_1^2 \quad n_2 \quad S_2^2 \quad d_2}$$

Where

$$d_1 \quad \bar{x} \quad \bar{x}\,, \quad \bar{x} \quad \frac{1}{n} \quad x$$

$$d \quad \bar{x} \quad \bar{x} \quad , \quad \bar{x} \quad \frac{1}{n} \quad x$$

$$and\ \bar{x} \quad \frac{n\ \bar{x} \quad n\ \bar{x}}{n \quad n} \quad combined\ mean$$

another formula is

$$S^2 \quad \frac{1}{n_1 \quad n_2} \quad n_1 S_1^2 \quad n_2 S_2^2 \quad \frac{n_1 n_2}{n_1 \quad n_2} \quad x_1 \quad x_2 \quad ^2$$

Where

$S \quad -\sum x \quad \bar{x} \quad , \qquad S \quad -\sum \quad x \quad \bar{x}$

2.4.4 COEFFICIENT OF VARIATION (CV)

It is sometimes desirable to compare several groups with respect to their relative homogeneity in instances where the groups have very different means. Therefore it might be somewhat misleading to compare the absolute magnitudes of the standard deviations. One might expect that with a very large mean one would find a fairly large standard deviation. One might therefore be primarily interested in the size of the standard deviation relative to that of the mean. This suggests that we can obtain a measure of the relative variability by dividing the standard deviation by the mean. The result has been termed the coefficient of variation, denoted by CV. Thus

CV = σ /X

Where σ is the SD and X is the mean.

The series which having lesser CV is more consistent than other

2.5 MEASURES OF ASYMMETRY

After studying the skewness and kurtosis, have an idea about the shape of the frequency curve of the distribution. A distribution is said to be symmetrical if the frequencies are on either side of the central value. It implies that both the right and left tails of the curve are exactly equal in shape and length. If a distribution is not symmetrical then it is called asymmetric or skewed in the direction of the extreme values, i.e., on the right – or on the left. Since extreme values give longer tail in its direction therefore, the distribution having longer right tail is called right skewed or positively skewed distribution. The left implies longer left tail. Thus a measure of skewness indicates the extent as well as direction of skewness of the distribution. The measure of kurtosis gives an idea whether the centre of the distribution is assuming flatness or peakedness similar to the hump of the normal probability curve or not.The means of skeweness are very useful in biological, chemical and physical laboratory works. They are also used in economic social statistics and medical statistics to study the behavior of the data.

2.5.1 SKEWNESS AND ITS MEASURES

Definition- By skewness of a frequency distribution we mean the degree of its departure from symmetry.

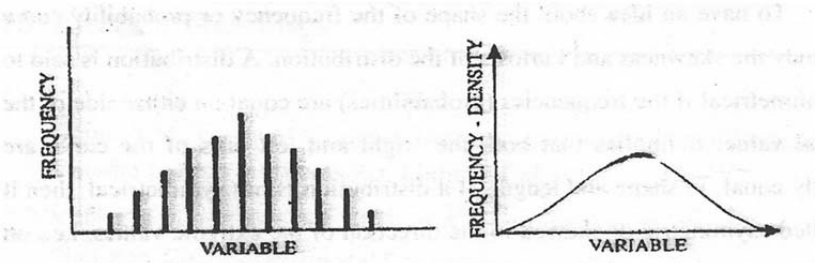


Fig. 2.1a A symmetrical distribution (discrete variable).

Fig. 2.1b A symmetrical distribution (continuous variable).

Figure 2.1a and 2.1b show two symmetrical distributions.

A distribution which is not symmetrical is called asymmetrical or skew. This skewness is said to be positive if the longer tail of the distribution is towards the higher values of the variable (Fig. 2.2a), negative if the longer tails is towards the lower values of the variable (Fig. 2.2b)

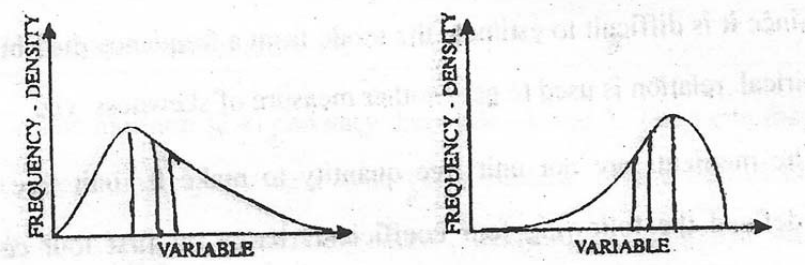


Fig. 2.2a A positively skew distribution.

Mode < Median < Mean

Fig. 2.2b A negatively skew distribution.

Mode < Median < Mean

For symmetrically shaped distribution: Mean = Median=Mode

For positively- skewed distribution: Mean > Median> Mode

For negatively skewed distribution: Mean < Median< Mode

Measures of Skewness: Pearsons Coefficient- An alternative measure of Skewness is obtained from the relative positions of the mean and the mode in a distribution.

In a symmetrical distribution, the mean, median and mode (assuming the distribution to be uni-modal) coincide. If the distribution is: skewed positively, then

mean> median>mode.

and if it is negatively skewed, then

mean<median<mode.

Hence the difference (mean mode) divided by the s.d., is taken as a measure of skewness.

$$Sk \quad \frac{\bar{x} \quad M}{s}$$

This is known as Peason's first measure of skewness, provided s>0.

another measure of skewness is.

$$Sk \quad \frac{3\ \bar{x} \quad M}{s}$$

Which is known as Pearson's second measure of skewness. If mean = median = mode then S_k=0. The limits can vary between -3 to 3.

Measures of Skewness: Bowley's Coefficient- For a symmetrical distribution the lower and upper quartiles are equidistant from the median; for a positively skew distribution the lower quartile is nearer the median than the upper quartile is, while for a negative skew distribution the upper quartile is nearer.

Thus the new measure based on quartiles is

$$Sk \quad \frac{Q \quad Mi \quad Mi \quad Q}{Q \quad Q} \quad \frac{Q \quad Q \quad 2Md}{Q \quad Q}$$

This is known as Bowley's measure of skewness. it has the limit -1 to 1.

2.5.2 KURTOSIS

Another method of describing a frequency distribution is to specify its degree of peakedness or kurtosis. Two distributions may have the same mean and the same standard deviation and may be equally skew, but one of them may be more peaked than the other.

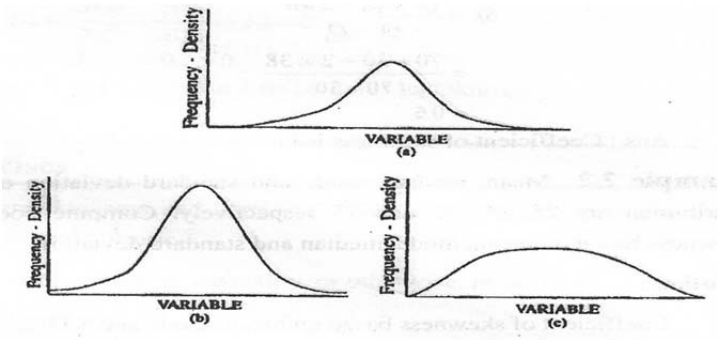


Fig. Three symmetrical distributions with different degrees of kurtosis : (a) mesokurtic, (b) leptokurtic, (c) platykurtic.

Concentration of values in the neighborhood of the central tendency and low tails, compared to a normal distribution with the same standard deviation. A normal curve is said to be mesokurtic (i.e. having medium kurtosis). A distribution with high peak is called leptokurtic, and one with flat peak is known as platykurtic.

Some Solved Examples of skewness and kurtosis-

Example- In a frequency distribution Q_1=30, Q_3=70 and median is 38, compute coefficient of skewness.

Solution- Coefficient of skewness based on quantities is

Sk ——— ——— 0.6

Answer- Coefficient of skewness is 0.6

Example- Mean, median and mode and standard deviation of frequency distribution are 25, 27, 35 and 15 respectively. Compute coefficients of skewness based on mean, mode, median and standard deviations.

Solution- Coefficient of skewness based on mean, mode and S.D. is

$$Sk \quad \frac{\bar{x} \quad M}{s} \quad \frac{25 \quad 35}{15} \quad \frac{10}{15} \quad 0.67$$

Coefficient of skewness based on mean, median and S.D. is

$$Sk \quad \frac{3\ \bar{x} \quad M}{s} \quad \frac{3 \quad 25 \quad 27}{15} \quad \frac{6}{15} \quad 0.4$$

2.6 MEASURES OF RELATIONSHIPS

Before reading this, blow some light on Bivariate Distribution and scattered diagram- Distribution involving two discrete variables is called a Bivariate distribution. For example, The height and weights of the students of a class in school.

Let (x_i, y_i), i =1,2,3,……..m; j=1,2,……,n, be a bivariate distribution. If the pair (x_i, y_i) occurs f_{ij} times then f_{ij} is called the frequency of the pair (x_i, y_i) and $\sum \sum f$ =N= the total frequency.

In general two variables are said to be related if they vary together in a systematic fashion. If two variables change in the same direction (either both increase or both decrease) they are said to be positively related. If the variables change in opposite direction (as one increases, the other decrease and vice versa), they are said to be negatively correlated. For example, The weight of an adult depends to some extent on the height.

Scatter Diagram and its Types- The simplest mode of diagrammatic representation of bivariate data is the use of *scatter diagram* (or dot diagram). A scatter diagram graphically summarizes the data on two quantitative variables by showing the joint distribution of the values on two variables. Each point in a scatter plot represents individual's scores on the two variables represented by the two axes of the graph.

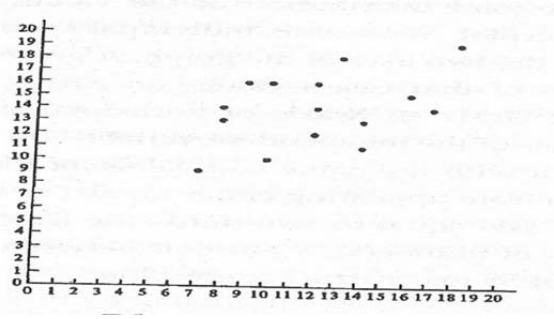
Example- A researcher is interested in studying the relationship between two variables the data are given below:

Case:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
X:	7	12	8	12	14	9	18	14	8	12	17	10	16	10	13

Y:	9	16	14	12	11	16	19	13	13	14	14	16	15	10	18

For these data, make a scattered diagram.

Solution- A scattered diagram of above data



Some more types of Scattered Diagrams are

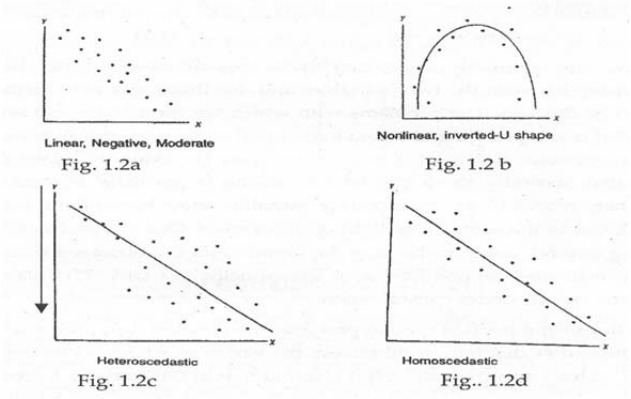


Fig. 1.2a Fig. 1.2 b

Fig. 1.2c Fig.. 1.2d

Sometimes from the scatter diagram, the variables are found to linearly related, at least approximately. If it is found that as one variable increases the other also increases, in general or on the average there will be said to be *positive correlation* between them. Following scatter diagrams shows the nature of correlation between two variables x and y.

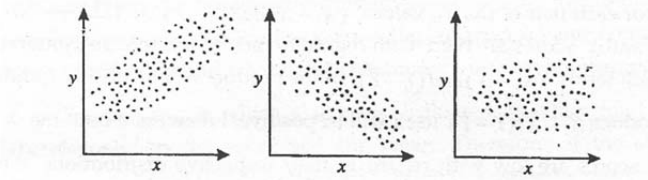


Fig-1. Fig.-2 Fig.-3

Positive correlation Negative Correlation Zero correlation

2.6.1 KARL PEARSON'S CORRELATION COEFFICIENT

The Pearson's correlation coefficient r_{xy} or product moment correlation coefficient is,

$$r \quad \frac{cov \quad X,Y}{\overline{var \quad X}\ \overline{var \quad Y}}$$

$$r = \frac{\frac{1}{N}\sum X X \quad Y Y}{\frac{1}{N}\sum X X \quad \frac{1}{N}\sum Y Y} \quad \frac{\sum X X \quad Y Y}{\overline{X X} \quad \overline{Y Y}}$$

$$r = \frac{\sum x x /N \quad XY}{X\sum /N \quad X \quad / \quad Y\sum /N \quad Y \quad /}$$

$$\frac{N\sum X Y \quad \sum X \quad Y\sum}{N\sum X \quad \sum X \quad / \quad N\sum Y \quad \sum Y \quad /}$$

Example Solution- The necessary information is presented in the following table. The first two columns contain the given X and Y score.

X_i	Y_i		X_iY_i	X_i^2	Y_i^2
7	9		63	49	81
12	16		192	144	256
8	14		112	64	196
12	12		144	144	144
14	11		154	196	121
9	16		144	81	256
18	19		342	324	361
14	13		182	196	169
8	13		104	64	169
12	14		168	144	196
17	14		238	289	196
10	16		160	100	256
16	15		240	256	225
10	10		100	100	100
13	18		234	169	324
Total	180	210	2577	2320	3050

Putting N=15, $\sum X$=180, $\sum Y$=210, $\sum X$ =2320, $\sum Y$ = 3050, $\sum XY$=2577 in equation (1.8) and (1.1), we get

$$r \quad \frac{N \quad \sum XY \quad \sum X \quad Y \sum}{N \quad \sum X^2 \quad \sum X^2 \quad N \sum Y^2 \quad \sum Y^2}$$

$$r \quad r \quad r \quad \frac{15\ 2577 \quad 180 \quad 210}{15 \quad 2577 \quad 180 \quad 15 \quad 2320 \quad 210} \quad 0.43$$

as a measure of degree of relationship between linearly related variable X & Y.

Properties of Correlation Coefficient- The properties of correlation coefficient r are:

(1) Correlation coefficient is a pure number having no unit.

(2) The limits of correlation coefficient r lies between -1 to +1, that is -1 r 1

(3) It is not affected by linear transformation of variables, that is, if u=X-A, v=Y-B, Then $r_{xy}= r_{uv}$ and is independent of A and B.

(4) If correlation coefficient between X and Y be r and regression coefficient b_{yx} and b_{xy}, then r = $\overline{b \quad b}$

(5) If two variables x and y are independent then r =0, but converse is not true.

Interpretation of values -1, +1 and 0 or r

(1) If ***r = +1***, it means that r.v. ensure perfect linear relationship between X and Y. In this case all points into scatter diagram lie on a straight line, extending form left bottom to the right bottom, if there is an increase in X then there will be proportional increase in Y.

(2) If ***r = -1***, it means that r.v. Y decreases with the increase in X and vice versa. There is perfect negative relationship between the variables. All the points in the scatter diagram (i.e. on a straight line extending from left top to right bottom).

(3) The value ***r = 0*** confirms the lack of linear relationship between two variables. All the points are scattered on the graph and hardly any these points lie in a straight line.

2.6.2 SPERMAN'S RANK CORRELATION COEFFICIENT

First, let us suppose that there is no tie, i.e., no two individuals are ranked equal in either variable. The ranks x's and y's take values 1, 2, 3,….. n in some order. Than spearman correlation coefficient is pronounced as Roh and defined as:

$$\rho \quad 1 \quad \frac{6 \sum d}{n \ n \quad 1}$$

This is Spearman's formula for the rank correlation coefficient. Since the rank correlation coefficient is simple product moment correlation coefficient between two series of ranks, it always lies between -1 to +1.

Rank Correlation Coefficient for Tied Ranks- If some of the individuals have the same rank in ranking they are said to be tied. If the same rank is allocated to m individuals, then we have a tie of length m.

So the Spearman's rank correlation coefficient in the case of tied ranks becomes

$$\rho \quad \frac{\frac{n \quad 1}{12} \quad \frac{T \quad T}{2} \quad \frac{1}{2n}\sum d}{\frac{n \quad 1}{12} \quad T \quad / \quad \frac{n \quad 1}{12} \quad T \quad /}$$

$$\rho \quad 1 \quad \frac{6 \ \frac{1}{n}\sum d \quad T \quad T}{n \quad 1}$$

Example- Ten hand writing were ranked by two judges in a completion. The ranking are given below. Calculate Spearman's rank correlation coefficient.

Hand writing										
	A	B	C	D	E	F	G	H	I	J
Judge-I	3	8	5	4	7	10	1	2	6	9
Judge-II	6	4	7	5	10	3	2	1	9	8

Solution- The differences of ranks between the ranks $d \quad x \quad y$ of two judges for ten observations are -3, 4, -2, -1, -3, 7, -1, 1, -3, 1

Hence

$$d \quad 9 \quad 16 \quad 4 \quad 1 \quad 9 \quad 49 \quad 1 \quad 1 \quad 9 \quad 1 \quad 100$$

Thus Spearman's rank correlation coefficient

$$\rho \quad 1 \quad \frac{6\sum d}{n \ n \quad 1} \quad 1 \quad \frac{6 \quad 100}{10 \quad 10 \quad 1} \quad 1 \quad \frac{600}{900} \quad 0.394 \quad Answer$$

Example- Two supervisors ranked 12 workers working under them in order of efficiency as follows.

No. of workers	1	2	3	4	5	6	7	8	9	10	11	12
Supervisor-I	5	6	1	2	3	8.5	8.5	4	7	11	10	12
Supervisor-II	5.5	5.5	2	2	2	9	7	4	8	10.5	12	10.5

Calculate rank correlation coefficient.

Solution- In the ranking, of first supervisor, there 15 one tie of length 2.

Thus

$$T \quad \frac{1}{12}\frac{2}{12}\frac{2}{} \quad 0.0417$$

In the ranking of second supervisor, there are three ties of length 2, 3 and 2 respectively. Thus

$$T \quad \frac{1}{12}\frac{2}{12}\frac{2}{} \quad \frac{3}{12}\frac{3}{} \quad \frac{2}{12}\frac{2}{} \quad 0.25$$

d	.25	.25	1	0	1	.25	2.25	0	1	.25	4	2.25	12.50

$$\rho \quad \frac{\frac{n}{12}\frac{1}{} \quad \frac{T}{2}\frac{T}{} \quad \frac{1}{2n}\sum d}{\sqrt{\frac{n}{12}\frac{1}{} \quad T \quad \frac{n}{12}\frac{1}{} \quad T}}$$

$$\frac{\frac{12}{12}\frac{1}{} \quad \frac{0.0417}{2}\frac{0.25}{} \quad \frac{12.50}{2 \quad 12}}{\sqrt{\frac{12}{12}\frac{1}{} \quad 0.0417 \quad \frac{12}{12}\frac{1}{} \quad 0.25}}$$

$$\frac{\cdot}{\sqrt{\cdot}\ \sqrt{\cdot}} \quad 0.956 \quad Answer.$$

2.7 REGRESSION ANALYSIS

The regression analysis is used for Prediction, which is an objective to all sciences, including social and behavior sciences. Prediction is based on relationship between or among variables. There are some types of regression analysis:

Simple regression- The regression analysis confined to the study of only two variables at a time is called the simple regression.

Multiple Regression- The regression analysis for studying more than two variables at a time is known as multiple regression.

Linear Regression- If the regression curve is a straight line, then there is a linear regression between the variables under study. In other words, in linear regression the relationship between the two variables X and Y is linear.

Non linear Regression- If a curve or regression is not a straight line, then it is called a non-linear or curvilinear regression.

Lines of Regressions- A line of regression is the line which gives the best estimate of one variable for any given value of the other variable.

2.7.1 LINE OF REGRESSION OF X ON Y.

It is the line which gives the best estimate for the values of X for a specified value of Y.

It is given by, $X \quad X \quad r— \; Y \quad Y$

Where X, Y are means of X series and Y series respectively are S.D. of X and Y series respectively and r is the correlation coefficient between X and Y.

2.7.2 LINE OF REGRESSION OF Y ON X.

It is the line which gives the best estimates for the values of Y for any specific values of X.

Regression equation of Y on X is given by:

$$Y \quad Y \quad r\frac{\sigma}{\sigma} \; X \quad X$$

It can also be put in the form

Regression Coefficients- The regression coefficient of Y on X is $b \quad r—$ and that of x on y is $b \quad r—$

Some Important Properties Relating to Regression Coefficient-

1. Numerically the correlation coefficient is the geometric mean of the two regression coefficients. As regards the sign of r, it is the same as the common sign of the two regression coefficients.

$$|r| \quad \overline{b \quad b}$$

2. If one of the regression coefficients is greater than unity, then the other is less than unity.

$$b \quad \frac{1}{b} \quad 1$$

3. Arithmetic mean of the regression coefficient is greater than the correlation coefficient.

4. Regression coefficients are Independent of change of origin but not of scale.

5. The sign of correlation is same as that of regression coefficients,

Example- Find both the regression equations from the following data:

$X \quad 60 \qquad Y \quad 40 \qquad XY \quad 1150$

$X \quad 4160 \qquad Y \quad 1720 \qquad N \quad 10$

Solution- The regression coefficient b_{xy} and b_{yx} given by:

$$b \quad \frac{\sum XY \quad \frac{\sum X \quad \sum Y}{N}}{\sum Y \quad \frac{\sum}{N}} \quad \frac{1150 \quad \frac{60 \quad 40}{10}}{1720 \quad \frac{40}{10}} \quad \frac{910}{1560} \quad 0.58$$

And

$$b \quad \frac{\sum XY \quad \frac{\sum X \quad \sum Y}{N}}{\sum X \quad \frac{\sum}{N}} \quad \frac{910}{4160 \quad \frac{60}{10}} \quad 0.24$$

Also

$$X \quad \frac{\sum X}{N} \quad \frac{60}{100} \quad 6; \qquad Y \quad \frac{Y}{N} \quad \frac{40}{100} \quad 4$$

Now, the regression equation of Y on X is-

$$Y \quad Y \quad b \quad X \quad X$$

$$\Rightarrow Y \quad 4 \quad 0.24 \quad X \quad 6 \quad or\, Y \quad 0.24X \quad 2.56.$$

The regression of X on Y is-

$$X \quad X \quad b \quad Y \quad Y$$

$$\Rightarrow X \quad 6 \quad 0.58 \quad Y \quad 4 \quad or\, X \quad 0.58Y \quad 3.68.$$

Example- From the following results, obtain the two regression equations and estimate the Y when the X is 29 . And the X, when the Y is 600:

	Y	X.
Mean	508.4	26.7
S.D.	36.8	4.6

Coefficient of correlation between yield and rainfall is + 0.52.

Solution-

Regression of y on x is given by $\quad y \quad y \quad r \quad x \quad \bar{x}$

Or

$$y \quad 508.4 \quad 0.52 \quad \frac{36.8}{4.6} \quad x \quad 26.7 \quad \Rightarrow y \quad 508.4 \quad 0.52 \quad 8 \quad x \quad 26.7$$

$\Rightarrow$ y - 508.4 = 4.16 x - 111.072 $\qquad \Rightarrow$ y = 4.16 x + 397.328

When x = 29, y = 4.16 **29 397.328 517.968**

Regression of x on y is given by $x \quad \bar{x} \quad r \quad y \quad y$

Or

$$x \quad 26.7 \quad 0.52 \quad \frac{4.6}{36.8} \; y \quad 508.4 \quad 0.065 \; y \quad 508.4$$

$$0.065 \; y \quad 33.046$$

Or x = 0.065 y - 6.346

When y = 600, x= 0.065 600 - 6.34 = 32.654

2.8 ASSOCIATION OF ATTRIBUTES

In real life we often study of characteristics/ qualitative data the numerical value are not possible. Simply we can observe the presence or absence of these specific characteristics (or properties) over a group of individuals or units under observation. Such characteristics are termed as attributes. Hence attribute is a qualitative character which cannot be measured numerically and one simply observes the presence or absence of it. Honesty, beauty, preferences, likings, colors, blindness, smoking etc are few examples of attributes. If an attribute is classified in two groups it is called a dichotomous attribute whereas if it is classified in many categories it is called manifold.

Combinations, Classes and Class Frequencies of Attributes (Dichotomous Classification)- Different attributes, their subgroups and combinations are called classes and the numbers of observation assigned to them are called respective class frequencies.

Example- Consider a dichotomous attribute "Smoking" which may be divided into two categories

1. Smokers (A)
2. Nonsmokers (α)

Let us further consider one more dichotomous attribute "Literacy" which is also classification into two categories

1. Literate (B)
2. Illiterate (β)

Both of these attributes are called dichotomous as they are divided into two subgroups and any individual will belong to only one of these categories either A or α and B or β.

Symbolically, we may write,

(AB) to denote the number of individuals who are smokers and literate.

(Aβ) to denote the number of individuals who are smokers and illiterate.

(α, B) to denote the number of individuals who are nonsmokers and illiterate.

(α, β) to denote the number of individuals who are non smokers and illiterate.

Thus in general (AB) stands for the frequency of the individuals or units which possess the attribute A and B simultaneously. Similar interpretations exits (α, β) for etc. Total number of observation is denoted by N.

Obviously we have,

$$N \quad A \quad \alpha \quad B \quad \beta$$

$$N \quad AB \quad A\beta \quad A\beta \quad \alpha\beta$$

Order of Class and Class Frequencies- Total number of observation N is taken as one class of order "Zero". (A), (B), (C), …… etc will be each of order one, $AB \quad A\beta \quad A\beta \quad \alpha\beta$ ….. etc. will be each of order three and so on. A class or class frequency (ABC…M, r letters) containing r attributes in it said to be of order "r".

With n attributes there are in all 2^n positive class frequencies.

With n attributes the number of negative class frequencies will be (2^n-1), because except N.

Consistency of Data- A data is said to be consistent if all of its class frequencies have been appeared to have been observed within one and the same population. Class frequencies of a consistent data will not have any mutual contradiction rather they will support each other.

Example- If, in a series of houses actually invaded by small pox 70% of the inhabitants are attacked and 85% have been vaccinated. What is the lowest % of the vaccinated that have been attacked?

	(B) Vaccinated	β Not vaccinated	
(A)Attacked	AB	$A\beta$	70
(α)Not attacked	βB	$(\alpha\beta)$	30
	85	15	100

Solution- Given that, (A)=70, (B) = 85, (α) = 30, (β) = 15, N = 100

Since, (AB) 0 and $(\alpha\beta)$ 0 $\Rightarrow$(1-A) (1-B) N 0

or (1 – A – B - AB) N 0

or N - (A) - (B) + (AB) 0

or (AB) (A) + (B) - N

or (AB) (70) + (85) - 100

or (/ \B) 55

So, out of 85 vaccinated at least 55 are being attacked.

Hence at least — 100 64.7% vaccinated have been attacked.

Contingency Tables- Let there be two attributes divided into two categories i.e. we are considering dichotomous classification attributes. In this case there are 9 class frequencies. These class frequencies in usual notations may be written in a 2 2 table as follows:

Attributes	A	α	Total
B	(AB) =a (say)	αB = b (say)	(B)
β	$A\beta$ = c (say)	$(\alpha\beta)$= d (say)	β
Total	(A)	α	N

This table is known as 2 2 contingency table. The contingency table need not be 2 2 only.

Independence and Association of Attributes- Consider two dichotomous attributes A and B.

So.

(A)= (AB) + $A\beta$

α = αB + $(\alpha\beta)$

(B) = (AB) + $(\alpha$B) (I)

$(\beta) = (A\beta) + (\alpha\beta)$ (2)

N = (AB) + (Aβ) + (αB) + (aβ)

Further,

N = (A) + (α) (3)

and

N = (B) + (β) (4)

For A and B to be independent we must have

$$\frac{AB}{A} \quad \frac{\alpha\beta}{\alpha} \quad \frac{B}{N}$$

Thus, (AB) = ——— then A and B are said to be independent.

If AB ———, then A and B are said to be positively associated.

If (AB) ——·——, then A and B are said to be negatively associated.

In general, if attributes are not independent they are said to be associated.

2.9 3 - SIGMA LIMITS

Three-sigma limits (3-sigma limits) is a statistical calculation that refers to data within three standard deviations from a mean. Three-sigma limits are used to set the upper and lower control limits. Mathematically it is defined as $\bar{x}$ 3σ. Here $\bar{x}$ 3σ is upper limit and $\bar{x}$ 3σ is lower limit.

2.10 SUMMARY

This unit gives detailed knowledge about Measures of central tendency, measures of dispersion, measures of symmetry, measures of relationships, predictions, association of attributes and 3-sigma limits, with numerical examples.

2.11 CHECK YOUR PROGRESS

1. A sample of 12 fathers and their eldest sons gave the following data about their heights in inches:

Father:	65	63	67	64	68	62	70	66	68	67	69	71
Son:	68	66	68	65	69	66	68	65	71	67	68	70

Calculation coefficient of rank correlation.

2. Calculate mean, median, mode and variance.

Class group (X)	*No. of observations(f)*
0-10	14
10-20	23
20-30	27
30-40	21
40-50	15
Total	100

3. Given that (A)= (α)=(B)= (β)= — show that (AB) = ($\alpha\beta$).

4. What are skewness and kurtosis? Give some suitable measures for skewness

5. Calculate Karl Pearson's coefficient of correlation for the data given below:

Independent Variable X	3	7	5	4	6	8	2	7
Dependent Variable Y	7	12	8	8	10	13	5	10

6. Find the two lines of regression from the following data:

Age of husband	25	22	28	26	35	20	22	40	20	18
Age of wife	18	15	20	17	22	14	16	21	15	14

Hence estimate:

i. the age of husband when the age of wife is 19 and

ii. The age of wife when the age of husband is 30.

2.12 FURTHER READINGS

1. Goon, Gupta & Dasgupta: Fundamentals of Statistics Vol. I the World Press Pvt. Ltd., Kolkata.

2. Yule G.U. and Kendall, M.G.: An Introduction to the Theory of Statistics, Charles Griffin and Company Ltd.

3. C.E. Weatherburn: Mathematical Statistics. Kenney, J.F and Keeping, E.S.: Mathematics of Statistics, Part I Van Nostrand, 1954 and Affiliated East- West Press.

4. Mills, F.C.: Statistical Methods (Ch.5) H. Holt, 1955.

5. Arora P. N., Arora Sumit. Arora A; Comprehensive Statistical Methods, S. Chand, New Delhi

6. Gupta S. C., Kapoor,V.K., Fundamentals of Mathematical Statistics, S. Chand, New Delhi.

Uttar Pradesh Rajarshi Tandon
Open University

MFN -107

Biostatistics

BLOCK

2

PROBABILITY-DISTRIBUTION THEORY AND DEMOGRAPHY

Course Design Committee

Dr. (Prof.) Ashutosh Gupta — **Director**
School of Science, UPRTOU Prayagraj

Prof. Umesh Nath Tripathi — **Member**
Department of Chemistry
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. S.I. Rizvi — **Member**
Department of Biochemistry
University of Allahabad, Prayagraj

Prof. Dinesh Yadav — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. Sharad Kumar Mishra — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Dr. Ravindra Pratap Singh — **Member**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Dr. Dharmveer Singh — **Member/Secretary**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Course Preparation Committee

Dr. Shruti — **Writer**
Sr. Assistant Professor, School of Sciences
U. P. Rajarshi Tandon Open University, Prayagraj

Prof. G. S. Pandey — **Editor**
Department of Statistics, University of Allahabad,
Prayagraj

Dr. Dharmveer Singh — **Course Coordinator**
Academic Consultant (Biochemistry)
School of Science, UPRTOU, Prayagraj

© **UPRTOU, Prayagraj - 2024**
ISBN - 978-93-94487-67-3

Printed and Published by Vinay Kumar, Registrar, Uttar Pradesh Rajarshi Tandon Open University, Prayagraj - 2024.

Printed by - K. C. Printing & Allied Works, Panchwati, Mathura -281003

BLOCK INTRODUCTION

The present SLM on Bio Statistics consists of three Blocks. *Block - 1 – Research Methodology and Statistical Methods* has two units; *Block - 2 – Probability - Distributions and Demography* has two units; and at the last *Block – 3 Tests of Significance and Analysis of Variance* has two units.

The Block-1– Research Methodology and Statistical Methods consists two units. The *first unit* of this block; named *Research Methods and Sampling Procedures*; describes the meaning and types of research, significance of research. It tells about the research problem and its selection with *Sampling Theory*; which discuss about the sampling, different types of sampling designs, simple random sampling, stratified sampling and cluster sampling with their applications.. The *second unit* of this block; named *Statistical Tools*; discusses about the measures of central tendency, measures of dispersion, measures of asymmetry, correlation and regression analysis, association of attributes and 3-sigma limits.

The Block-2– Probability- Distributions Theory and Demography is the second block having two units. The *first unit* of this block; named *Probability and Distribution Theory*; gives the Basic concepts of probability, definitions of probability, additive and multiplicative law of probability, conditional probability, Bayes' theorem, random variable and its types, probability mass function and probability density functions. In *Probability Distributions*, the concept of probability distribution, discrete and continuous probability distributions namely Binomial Distribution, Poisson Distribution, Geometric Distribution, Normal Distribution, Exponential Distribution have been also discussed in this unit along with their properties, applications and importance.. The *second unit* of this block; named *Demography*; gives knowledge about the vital statistics and demography, this also tells about the source of vital statistics and demographic data, rates, ratio, proportion, measures of fertility, measures of mortality, measures of morbidity and migration.

The Block-3– Tests of Significance and Analysis of Variance consists of two units. The *first unit* of this block; named *Testing of Hypothesis*; discuss about the hypothesis and its types, level of significance, critical region, p-value, types of errors, chi-square tests, t-tests and z-tests with their applications. The *second unit* of this block; named *Analysis of Variance*; discusses about the concept of analysis of variance and co-variance, basic principles of ANOVA and ANCOVA. (One Way, Two Way and Three Way Analysis).

Illustrations and examples on these topics have also been given.

At the end of every block/unit the summary, self assessment questions and further readings are given.

UNIT-3 PROBABILITY AND DISTRIBUTION THEORY

Structure

3.1 INTRODUCTION

In various fields of social, biological physical sciences etc., we come across with experiments and phenomenon in which some kind of uncertainty is involved. This uncertainty leads to the study of Probability. This unit introduces the basic idea of such approach to probability will be explained and their distributions will be discussed.

3.2 OBJECTIVES

After going through this unit you shall be able to:

- Understand the probability and its laws/properties.
- Understand probability function and its types.
- about probability distributions

3.3 DEFINITION AND LAWS OF PROBABILITY

Before study the definition of Probability, the information of these given terms are important.

- The experiment is termed as *Trial* and the results are *Events* or *Cases*, e.g. tossing of a coin is a trial and getting head or tail is event.
- The total number of all possible results of any trial is *Exhaustive cases*, e.g. in above example there are only two exhaustive cases.
- When the happening of one event affects the happening of another, is *Mutually Exclusive*. In other words both events cannot occur at the same time, e.g. in above example head or tail is mutually exclusive events.
- When the result of one event does not affected by the other, is *Independent events*, e.g. if a coin tossed twice, the result of both throw is not affected to each other.
- when the happening of every event is same, called *Equally Likely events* e.g. in above example the occurrence of head or tail is equally likely.

Definition- If an event can occur in n total number of exhaustive/ mutually exclusive/equally likely cases and m of them are in favorable to the happing of a particular event, say A, then the probability of occurrence of the event A is given by

$$P\ A \quad p \quad \frac{Favourable\ number\ of\ cases}{Exhaustive\ number\ of\ cases} \quad \frac{m}{n}$$

p is known as probability of success and probability of failure is q.

$$q \quad 1 \quad p \quad 1 - \quad ——$$

Hence $p \quad q \quad 1$ And $0 \quad p, q \quad 1$

Hence the mathematical value of p is lies between zero to unity (one). It may not be negative. If the mathematical value of p is 0, then related event is called impossible event and if the mathematical value of p is 1, then related event is called sure possible event.

Laws of Probability- On the basis of the above definition of the probability some rules are important to know:

3.3.1 ADDITIVE LAW OF PROBABILITY

If $A_1, A_2 \ldots\ldots\ldots\ldots\ldots A_n$ are the mutually exclusive events then

$$P\ A \ \text{or}\ A \ \text{or} \ldots\ldots\ldots \text{or}\ A \qquad P\ A \qquad P\ A \qquad \cdots\ldots\ldots\ldots \quad P\ A$$

If $A_1, A_2 \ldots\ldots\ldots\ldots\ldots A_n$ are the equally likely events then the sum of their probabilities is unity. i.e.

$$P\ A \qquad P\ A \qquad \cdots\ldots\ldots\ldots \quad P\ A \qquad 1$$

In other words

For two mutually exclusive events A and B, ($A \cap B = \emptyset$).

$$P(A \cup B) = P(A) + P(B)$$

For pair wise mutually exclusive events $A_1, A_2, \ldots\ldots\ldots$ $A \cap A \qquad \emptyset\ \forall\ j \ ,$

$$P \qquad A \qquad A$$

3.3.2 MULTIPLICATIVE LAW OF PROBABILITY

If $A_1, A_2 \ldots\ldots\ldots\ldots\ldots A_n$ are the independent events then

$$P\ A \ \text{and}\ A \ \text{and} \ldots\ldots\ldots \text{and}\ A \qquad P\ A \qquad P\ A \qquad \ldots\ldots\ldots\ldots \quad P\ A$$

This is called ***Multiplicative law of Probability*.**

3.4 CONDITIONAL PROBABILITY

Definition- Let A, B be two events defined on the same sample space Ω. Then conditional probability of B given A denoted by P(B|A), is defined as

$$P(B|A) = \frac{\cap}{\qquad}, \quad provided\ P \quad A \quad 0.$$

The conditional probability satisfies all the rules of a probability.

Definition- Two events A and B are said to be independent whenever

$P(A \cap B) = P(A)\ P(B)$

Note- If A and B are independent events then P(B|A) = P(B) and P(A|B) = P(A), provided

$P(A) > 0, P(B) > 0.$

3.5 BAYE'S THEOREM

Let $B_1,\ldots\ldots\ldots\ldots,B_k$ be a partition of the sample space Ω and A be an event ($A \subset \Omega$), Then

$$P\ B\ |A \qquad \frac{P\ A|B\ \ P\quad B}{\sum_{\ 1} P\quad A\quad BP\quad B}$$

Proof- We have

$$P\ B\ |A \qquad \frac{\cap \qquad |}{}$$

Where the last equality follows form the multiplicative law of probability. Further, from the previous result, we have,

$$P\ A \qquad P\quad A\quad BP\quad B$$

$$1$$

Substituting the value of P(A), we obtain

$$P\ B\ |A \qquad \frac{|}{\Sigma\ _1}$$

Hence the result

Note- Here the probabilities $P(B_j)$'s are known as **"a priori (or prior) probabilities"**. They exist before we gain any information about A (the result of an experiment). The probabilities $P(A|B_j)$'s are known as "**likelihoods".** They indicate how likely event A to occur under the information that B_j occurs.The probabilities $P(B_j|A)$'s are known as '**a posteriori (or posterior) probabilities".** They are determined after the results of the experiment are known.

Example- A bag contains 10 coins out of which five coins of type 1 are unbiased and remaining five coins are biased. Among the biased coins, four coins of type II have probabilities of head 1/3 are remaining one coins of type III have probability of head 9/10. A coin is selected at random and tossed three times. If we get heads in a row, what is the probability that the coin is of type III.

Solution- Let

B_1 : Event the coin is of type I

B_2 : Event the coin is of type II

B_3 : Event the coin is of type III

A: Event that we get three heads in a row.

Given that $P(B_1) = 0.5$ $P(B_2) = 0.4$, $P(B_3) = 0.1$

$$P\ A|B \qquad 1/2\ ^3,\ P\ A|B \qquad 1/3\ ^3, P\ A|B \qquad 9/10\ ^3$$

Then the probabilities that the coin is of type III given that three heads have been obtained in a row, i.e., $P(B_3|A)$, is given by

$$P(B_3|A) = \frac{P(A|B)\,P(B_3)}{P(A|B)\,P(B_1) + P(A|B)\,P(B_2) + P(A|B)\,P(B_3)}$$

$$= \frac{\frac{9}{10}^{3} \; 0.1}{\frac{1}{2}^{3} \; 0.5 + \frac{1}{3}^{3} \; 0.4 + \frac{9}{10}^{3} \; 0.1} = 0.46$$

3.6 RANDOM VARIABLES AND PROBABILITY FUNCTION

A random variable is a variable whose value is unknown or a function that assigns value to each of an experiments result. It is used to measure outcomes of random occurrences in probability theory.

There are two types of random variables; discrete and continuous.

3.6.1 DISCRETE RANDOM VARIABLE AND PROBABILITY MASS FUNCTION

A random variable X is said a discrete random variable if it takes finite or countable infinite number of values. Thus the range of a discrete random variable has countable number of points.

Let X be a discrete random variable with $x_1, x_2, \ldots\ldots x_i$, possible outcomes than $P(x_i) = P(X = x_i)$; i= 1,2,…. Satisfying the following conditions:

(i) $p(x_i)$ 0 for all i = 1, 2,…..

(ii) $\sum \; p \; x \; 1$

Then the function p(x) satisfying the above conditions is called the ***probability mass function*** (pmf) of the random variable X.

The collection of pairs (xi, p(xi)); i = 1, 2…. is called the probability distribution of X. the cdf of X is given by

$$F\ x \quad i \quad p \quad x$$

,

3.6.2 CONTINUOUS RANDOM VARIABLE AND PROBABILITY DENSITY FUNCTION

A random variable X is said a continuous random variable if it takes infinite number of values. Thus the range of a continuous random variable has uncountable number of points.

For continuous random variables, there exists a function f(x) called the ***probability density function*** (pdf) of X, such that

(i) f(x) 0 for all x.

(ii) $f\ x\ dx \quad 1$

(iii) For real number a, b with $\infty < a < b < \infty$, the probability that X lies in interval (a,b] is given by

$$P\ a \quad X \quad b \quad fx\ dx$$

Since for a continuous random variable X, the probability of a single point P(X = x) = 0 for all x, we have

$$P\ a \quad X \quad b \quad Pa \quad X \quad b \quad P \quad a \quad X \quad b$$

The cdf of X is given by

$$F\ x \quad PX \quad x \quad f \quad dxx$$

Conversely, the pdf of X, in terms of cdf, is given by

$$f\ x \quad \frac{d}{dx} F \quad x$$

3.7 PROBABILITY DISTRIBUTION

There are two types of probability distributions: discrete and continuous.

3.7.1 DISCRETE DISTRIBUTIONS

The distribution functions which are based on discrete random variables is known as discrete distributions. Here we study about the binomial distribution, Poisson distribution and Geometric distribution.

3.7.1.1 BINOMIAL DISTRIBUTION

Definition- The binomial distribution with parameter n, p distribution of random variable X for which,

$$P\ X \quad x \quad \binom{n}{x} \quad pq \qquad\qquad x \quad 0, 1, 2, \ldots\ldots, n$$

$$\binom{n}{x} \quad pq \qquad \frac{n!}{x!\ \ n \quad x\ !} p\ \ q$$

Note-

- The mean of binomial distribution is np and variance is npq.
- The sum of independent binomial variates is not a binomial variate. In other words, binomial distribution not holds the additive or reproductive property.

Example- Ten coins are throwing simultaneously. Find the probability of getting at least eight heads.

Solution- p = probability of getting a head = 1/2

q = probability of not getting a head = 1/2

The probability of getting x heads in a throw of 10 coins is

$$p\ x \qquad \binom{10}{x}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \qquad \binom{10}{x}\left(\frac{1}{2}\right) \quad .x \qquad 0, 1, 2, \ldots\ldots, 10$$

∴ Probability of getting x heads is given by

P[x 8] = P(8) + P(9) + P(10)

$$\frac{1}{2} \qquad \binom{10}{8} \qquad \binom{10}{9} \qquad \binom{10}{10} \qquad \frac{45 \quad 10 \quad 1}{1094} \qquad \frac{56}{1024} \qquad \frac{7}{128}$$

Example 2- The probability that a person recovers from a serious decrease is 0.30. Find the probability that at least one of 5 persons admitted to a hospital will survive.

Solution- If X denotes the number of persons recovered from decrease. Then we have to find P[X∼1]. X has binomial distribution with n = 5, p = 0.30.

We have

$$P\ X \qquad 1 \qquad 1 \qquad Px \qquad 0$$

$$1 \qquad \binom{5}{0} \qquad 0.30 \qquad 0.70 \qquad 1 \qquad 0.168 \qquad 0.832$$

Example- Comment on the following:

The mean of a binomial distribution is 4 and variance is 5.

Solution- If the given binomial distribution has parameter n and p then we have

mean = n p = 4 and variance = n p q = 5

Dividing eqn. we get

q= 5/4

which is impossible, since probability cannot exceed unity. Hence given statement is wrong.

Example- The mean and variance of binomial distribution are 3 and 3/4 respectively. Find P[X 1].

Solution-

Mean = E(X) = nq = 3

Var (x) = npq = 3/4

Dividing we get q = 1/4 ∴ p –

Now we have n p = 3

$$\Rightarrow \frac{3}{4}n \quad 3$$

$$\Rightarrow n \quad \frac{4}{3} \quad 3 \quad 4$$

$$\therefore P\ X \quad 1 \quad 1 \quad PX \quad 0 \quad 1 \quad q$$

$$1 \quad \frac{1}{4} \quad 1 \quad \frac{1}{256} \quad \frac{255}{256} \quad 0.996$$

3.7.1.2 POISSON DISTRIBUTION

The probability distribution having p(x) = p(x, λ) = $\frac{G}{!}$, Where λ > 0 is a constant; as its probability mass function is known as Poisson distribution with parameter λ.

This distribution was discovered by Simen Denis Poisson in 1837. Poisson distribution is a limiting case of binomial distribution under the following conditions:

(i) n, the number of trails is indefinitely large, i.e., n→ ∞.

(ii) p, the constant probability of success for each trial is indefinitely small i.e., p→ 0.

(iii) np = λ (say), is finite. Thus $- and\ q \quad 1 \quad -$, where is a positive real number.

Note-

- The mean and variance of Poisson Distribution is λ.
- Sum of independent Poisson variates is also a Poisson variate

The following are some examples of Poisson Variate:

(i) The number of defective screws per box of 100 screws.

(ii) The number of printing mistakes at each page of a book.

(iii) The number of deaths in a district in one year by rare disease.

(iv) The number of air accidents in some unit of time.

(v) The number of suicides reported in a particular city.

(vi) The number of cars passing through a certain street in time.

3.7.1.3 GEOMETRIC DISTRIBUTION

Definition-A random variable X is said to have the geometric distribution with parameter p, 0 < p < 1, if its p.m.f. is given by

$$P \quad X \quad X \quad \begin{matrix} q & p, & x & 0,1,2,3 \ldots \ldots \\ 0 & & other\ wise & \end{matrix}$$

Note-

- The mean = 1/p and V(X) = q/ p^2, hence for geometric distribution variance is greater than mean.

3.7.2 CONTINUOUS DISTRIBUTION

The distribution functions which are based on continuous random variables is known as continuous distributions. Here we study about the normal distribution and exponential distribution.

3.7.2.1 NORMAL DISTRIBUTION

It has been observed that most business and economic variables result in continuous data whose behavior is often best described by a bell-shaped continuous curve. Since most populations on these variables normally yield a bell-shaped curve, such a curve has come to be universally known as a normal curve. Accordingly, the probability distribution described by normal curve is called the normal (probability) distribution.

The normal distribution is defined by the probability density function.

$$f\ x \quad \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\quad}{\quad}} \quad ;for \quad \infty \quad x \quad \infty \qquad \ldots\ldots\ldots\ldots \quad 3.1$$

Where r, σ and > are two parameters of the distribution.

A continuous r.v. X having above pdf is called normal random variable. We write it as X~N (μ, σ) μ and σ are the parameters of the distribution. We shall see μ that is mean and σ is variance of the distribution.

This curve is bell-shaped and symmetric about mean μ., As the curve is symmetrical about the ordinal at x = μ, The curve is concave down wards in the centre μ, but after x = μ σ is becomes concave upwards. The two points of inflexion are given by x = μ σ,

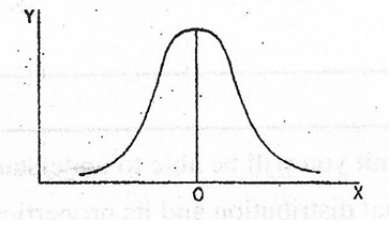


Figure: 3.1 Normal Distribution

It shows the deviations of the values of normal variables X from its mean μ. The larger these deviations, the higher are the value of standard deviations σ (or variance σ) which is the denominator in the exponent.

The shape of the curve shows that the observations occur most frequently in the neighborhood of the mean and their frequency decreases as they move away from the mean. From (3.1) it is seen that the distribution is symmetrical about the point $x \quad \mu . f\mu \quad a \quad f\mu \quad a \quad \frac{}{\sqrt{\quad T}} exp \quad \sigma \quad a \quad .$

Whatever μ may be. Hence μ is mean as well as median of the distribution. Againμ is also mode of the distribution, since $f \ \mu \quad 0 \ and \ f \quad \mu ko.$ Also exp — decreases monotonically$a$ as increase from zero, i.e., as a deviates from zero in either direction.

Thus the mean, median and mode of the distribution coincide at μ, Thus the mode μ of the distribution also coincides with mean and median μ.

Approximately 68% of the area under the cure lien in the region [$\mu \quad \sigma$, $\mu \quad \sigma$], approximately 95% in [$\mu \quad 2\sigma, \mu \quad 2\sigma$] and almost all in the region [$\mu \quad 3\sigma, \mu \quad 3\sigma$]. In fact approximately 0.27% area lies outside region [μ $3\sigma, \mu \quad 3\sigma$]. This is Area Property.

Normal Distribution with mean μ and variance σ is also denoted by N (μ, σ) or N (μ, σ) Mean, Median and Mode of this distribution, coincide at theμ.

Standard Normal Distribution- If X~N (μ, σ) then its pdf is given by equ. (3.1) using transformed

$$z \quad \frac{x \quad \mu}{\sigma}$$

We obtain the pfd of as

$$\varphi \ z \quad \frac{1}{\sqrt{2\pi}} e^{\ -} \qquad for \quad \infty \quad z \quad \infty \qquad 3.1a$$

Obviously E(z) = 0 and var (Z)=1

Hence, z is standard normal variable and the pdf given by equ. (3.1a) is called standard normal distribution.

Properties of Normal Distribution-

- The normal distribution is symmetrical about X = μ i.e., there is no skewness in normal distribution
- The normal distribution is mesokurtic i.e., Kurtosis of a normal distribution vanish; it is neither playtokurtic nor leptokurtic.
- Any linear function of normal variate is also normally distributed.

Properties of a Normal Curve-

(i) Normal curve is symmetric; bell shaped and unimodal.

(ii) The mean, median and mode all coincide.

3.7.2.2 EXPONENTIAL DISTRIBUTION

Exponential distribution plays important role in statistics. It has been used as potential model for lifetimes of many things. Exponential distribution is also used for continuous waiting time random variable of various events whereas geometric distribution is used for discrete waiting time random variable.

Definition- A continuous random variable X is said to follows an exponential distribution if it assumes non negative values with probability density function (p.d.f.) given by

$$f(x,0) = \begin{cases} \theta e^{\quad}, & x \quad 0,\theta \quad 0 \\ 0, & otherwise \end{cases}$$

Here θ is the parameter of exponential distribution

Note-

- Mean and variance of the exponential distribution is

$$\begin{matrix} Mean & \frac{1}{\theta} \\ Variance & \frac{1}{\theta} \end{matrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4.15$$

3.8 SUMMERY

This unit gives a complete idea about probability, random variables, its types, probability distributions, discrete and continuous distributions.

3.9 CHECK YOUR PROGRESS

1. Discuss about the mean and variance of Binomial Distribution.
2. Is mean and variance of Poisson distribution is equal.
3. In geometric distribution, is mean is greater than variance?
4. Draw the curve of normal distribution.
5. Find the variance of exponential distribution?

3.10 FURTHER READINGS

1. Cramer H, Mathematical Methods of Statistics, Princeton University Press, 1946 and Asia Publishing House, 1962.
2. Hogg R.V. and Craig A.T., Introduction to Mathematical Statistics, Macmillan, 1978.
3. Prazen E., Modern Probability Theory and its Applications, John Wiley, 1960 and Wiley Eastern 1972.

4. Rao C.R., Linear Statistical Inference and Its Applications, John Wiley, 1960 and Wiley Eastern 1974.

5. Rohtagi V.K. (1984), An Introduction to Probability Theory and Mathematical Statistics, John Wiley, 1976 and Wiley Eastern 1985.

6. Vikas S.S., Mathematical Statistics, John Wiley, 1962 and Toppan.

7. Arora P. N., Arora Sumit. Arora A; Comprehensive Statistical Methods, S. Chand, New Delhi

8. Gupta S. C., Kapoor,V.K., Fundamentals of Mathematical Statistics, S. Chand, New Delhi.

UNIT-4 DEMOGRAPHY

Structure

4.1 INTRODUCTION

The events happening in human's life are called *vital events* as births, deaths, migration, marriage, divorce, adoptions, separations, etc. and the methods

and techniques which used in the analysis of such data are termed as *vital statistics.* It is used by the any government for population estimation and projection and trends of population and for developing the overall planning and evaluation of economic and social development programmes. For collecting such data the registration method, census enumeration and sample survey etc. are some popular methods.

4.2 OBJECTIVES

After going through this unit you shall be able to:

- Understand the concept of demography.
- Understand measures of fertility and mortality.

4.3 RATES, RATIO AND PROPORTION

A ratio compares the extent of two quantities. When the quantities have different units, then this ratio is called a rate. Mathematically, If a and b are two quantities f the same kind (in same unit) than the fraction a/b is called the ratio of a to b= a/b or a:b.

A proportion is a statement of equality between two ratios. Mathematically, a, b, c and d are said to be in proportion if a:b=c:d

i.e. if a/b=c/d i.e. if ad=bc

this proportion is also written as a:b::c:d

4.4 FERTILITY AND MEASURES OF FERTILITY

Fertility is a natural capability to deliver the children (live births). Generally this study is related with the effects of contraception, post partum abstinence and breastfeeding on fertility and the birth interval also play role in many other population measures.

Some measurements of fertility are as follows:

4.4.1 CRUDE BIRTH RATE (CBR)

It is simple, easy to calculate and easy to understand. But it is not a reliable estimate, because it considers overall population, male female in all age group which is not true. The mathematical formula is

$$CBR \quad \frac{\text{Total number of live births}}{\text{Total population}} \quad 1000$$

4.4.2 GENERAL FERTILITY RATE (GFR)

It is simple, easy to calculate and easy to understand. In order to overcome of the drawback of CBR, it considers the females in the reproductive age. The

mathematical formula is

$$G.F.R.$$

$$\left\{\frac{\begin{array}{c}Total\ number\ of\ live\ births\ which\\ occurred\ among\ the\ population\\ of\ a\ region\ during\ a\ given\ year\end{array}}{\begin{array}{c}Mid\quad year\ female\ population\ of\\ reproductive\ ages\ 15\ to\ 49\ in\ the\\ given\ area\\ during\ the\ same\ year\end{array}}\right\}\ 1000$$

4.4.3 AGE SPECIFIC FERTILITY RATE (ASFR)

In order to overcome of the drawback of GFR, it considers the live births with respect to the females in the different age group of reproductive age. The mathematical formula is

$$A.S.F.R.$$

$$\left\{\frac{\begin{array}{c}Annual\ live\ births\ which\\ occurred\ to\ a\ specified\ age\ group\\ of\ the\ female\ population\ of\ a\ region\\ during\ a\ given\ year\end{array}}{\begin{array}{c}Mid\quad year\ annual\ female\ population\\ of\ same\ specified\ age\quad group\\ in\ the\ given\ area\\ during\ the\ same\ year\end{array}}\right\}\ 1000$$

4.4.4 TOTAL FERTILITY RATE (TFR)

It is the most important measurement of fertility. The total fertility rate is the sum of the age specific fertility rates from a given age to the last point of child bearing age of a female. Symbolically,

$$TFR \qquad ASFR$$

4.5 MORTALITY AND MEASUREMENTS OF MORTALITY

Population size may decrease due to death. The numbers of deaths in a given period are known as mortality. Some measurements of mortality are as follows:

4.5.1 CRUDE DEATH RATE (CDR)

It is simple, easy to calculate and easy to understand. But it is not a

reliable estimate, because it ignores age and sex of the population. Mathematically it is defined as

$$CDR \quad \frac{Total\ Annual\ Deaths}{Total\ Annual\ Population} \quad 1000$$

The crude death rate can be calculated for males and females separately. It is usually lies between 8 and 30 per 1000. The female rate is generally lower than the male rate.

4.5.2 SPECIFIC DEATH RATE (SDR)

It is find the mortality experience in different sections of the population such as infants, mothers, male etc. The formula is:

$$SDR$$

$$\left(\frac{\begin{array}{c} Total\ deaths\ which\ occurred\ in\ among\ a \\ specific\ group\ of\ the\ population \\ of\ a\ given\ area \\ during\ a\ given\ period \end{array}}{\begin{array}{c} Mid\ year\ population\ of\ the\ specific \\ group\ of\ the\ population \\ in\ the\ same\ area\ during \\ the\ same\ period \end{array}} \right) \quad 1000$$

4.5.3 AGE SPECIFIC DEATH RATE (ASDR)

In order to overcome of the drawback of CDR, it considers the age of the population. The mathematical defined as

$$ASDR$$

$$\left\{ \frac{\begin{array}{c} Annual\ deaths\ which \\ occurred\ to\ a\ specified\ age \\ group\ of\ the\ population \\ of\ a\ region\ during\ a\ given\ year \end{array}}{\begin{array}{c} Mid\ \ year\ annual\ population \\ of\ same\ specified\ age\ \ group \\ in\ the\ given\ geographical\ area \\ during\ the\ same\ year \end{array}} \right\} \quad 1000$$

4.5.4 INFANT MORTALITY RATE (IMR)

It is a death rate of kids who are less than one year of age. It is an important indicator of overall physical health of community. If any community has high IMR it means there is a need of more medical care nutrition etc.

$$IMR \quad \frac{No.of\ deaths\ under\ 1\ year\ of\ age}{No.of\ live\ birth} \quad 1000$$

4.6 MEASUREMENT OF POPULATION GROWTH

The fertility and mortality rates separately both are not enough to give the idea about the rate of population growth. So, for obtaining the rate of growth of population these measures are useful.

4.6.1 CRUDE RATE OF NATURAL INCREASE (CRNI)

It is simple and easy to understand. Formula is

$$CRNI \quad CBR \quad CDR$$

4.6.2 CRUDE RATE OF NATURAL INCREASE (CRNI)

It is throws light on the likely growth of population. Formula is

$$PVI \quad \frac{CBR}{CDR} \quad 100$$

4.6.3 GROSS REPRODUCTION RATE (GRR)

It is the sum of the age sex specific fertility rates (ASSFR) calculated from female births for each year of reproductive period.

$$GRR$$

$$\left\{ \frac{\begin{array}{c} Annual\ live\ female\ births \\ which\ occurred\ to\ a \\ specified\ age\ group\ of \\ the\ female\ population \end{array}}{\begin{array}{c} population\ of\ same\ specified \\ Mid \quad year\ annual\ female \\ age\ group \end{array}} \right\} \quad 1000$$

$$GRR \qquad ASSFR$$

The GRR is computed by the following formula also:

$$GRR \quad \frac{No.of\ female\ births}{Total\ no.of\ births} \quad TFR$$

The value of GRR is always lies between 0 to 5.

4.6.4 GROSS REPRODUCTION RATE (GRR)

GRR excludes the effect of the mortality on the birth rate. But NRR consider the mortality also. Symbolically,

$$NRR \quad \sum \quad female\ ASSFR \qquad surival\ factor \qquad 1000$$

NRR lies between from 0 to 5 and $NRR \quad GRR$

If NRR is equals to 1 the population is constant means all female babies exactly replaced her mother up to the reproductive period. If NRR is less than 1 indicates the declining population and if it is greater than 1 shows the increasing population. It is used for population projection.

4.7 MORBIDITY

Morbidity is the condition of being ill, diseased, or unhealthy. The example of an acute illness can be the flu, a broken arm, or a heart attack. Chronic illnesses are more like diabetes, or cancer. A person can live for several years with one or more morbidities which refer to having a disease or a symptom of disease, or to the amount of disease within a population. Morbidity also refers to medical problems caused by a treatment.

Example- Determination of Gross and Net Reproduction Rates for India, 1993

(1) Age in Year	(2) Age-specific fertility rate	(3) Female life-table stationary population	(4) Col. (2) x col. (3)
15-19	0.0696	4180	290.9
20-24	0.2346	4123	967.3
25-29	0.1897	4063	770.8
30-34	0.1143	4001	457.3
35-39	0.0611	3934	240.4
40-44	0.0285	3860	110.0
45-49	0.0101	3763	38.0
			2,874.7
Total	0.7079	-	

The sex ratio at birth for the country may be supposed to be 105 males to 100 females. Hence the above table, we get

$$GRR \quad 5 \quad 0.7079 \quad \frac{100}{205} \quad 1.73$$

$$and\ NRR \quad \frac{2874.7}{1000} \quad \frac{100}{205} \quad 1.40$$

Example- The procedure for computing the GRR and NRR is also shown in the following Table:

Methods for Computing GRR and NRR, U.P. 1971

Age group	Female population (1)	Live Birth (2)	ASFRs (per women) (3)=(2)/(1)	Female live birth (4)	Age specific maternity rates (5)=(4)/(1)	Age-specific survival rates* (5Lx/l0) (e =68) (6)	Expected Female births per women (7)= (5)x(6)
15-19	79865	8813	0.1103	4293	0.0538	0.90530	0.0487
20-24	63315	7620	0.2783	8582	0.1355	0.90048	0.1220
25-29	51680	4585	0.2812	7104	0.1370	0.89472	0.1226
30-34	44440	0235	0.2303	4986	0.1122	0.88799	0.0996
35-39	38795	569	0.1951	3686	0.1950	0.88009	0.0836
40-44	32250	760	0.058	1344	0.0417	0.87013	0.0363
45-49	26720	81	0.0143	186	0.0070	0.85705	0.0060
Total	-	1963	1.1951	30181	0.5822	-	0.5188
TFR GRR* NRR	- - -		5,9755 - -	- - -	2.9110	- - -	- 2.5940 -

Here

$$NRR \quad \frac{1}{1} \quad L \quad f \quad .5188 \quad 5 \quad 2.60$$

* Probability of surviving from birth to the midpoint of the age group,

$$** \, GRR \quad TFR \quad \frac{1}{1 \quad S.R.at\ birth}$$

$$5.9755 \quad .4878 \quad 2.91$$

Or

$$GRR \qquad 5f \qquad .5822 \qquad 5 \qquad 2.91$$

$$NRR \qquad 5f \qquad \frac{5l}{l} \qquad .5188 \qquad 5 \qquad 2.91$$

4.7 SUMMERY

In this unit we studied about the vital statistics and its all measures

4.8 CHECK YOUR PROGRESS

1. The quinquennial fertility rates (computed on the basis of female births alone) for Kerala 1961 are shown in the following table, together with the survival factor for each 5 year age-group (which is the probability for a newborn female to survive till the mid point of the age group and is approximately equal to $f\ L\ /5\ l$):

Age	Fertility rate (female births)	Survival factor
15-19	0.0106	0.968
20-24	0.0660	0.968
25-29	0.0673	0.964
30-34	0.0410	0.957
35-39	0.0214	0.953
40-44	0.0065	0.944
45-49	0.0006	0.929

Compute the GRR and NRR for Kerala for 1961 on the basis of the above data.

2. The number of births occurring in Assam in 1988 is shown here classified according to age of mother, together with the female population in each age-group of the child bearing period:

Age	Female Population	Number of births to mothers in the age group
15-19	200	4,000
20-24	173	26,000
25-29	161	32,000
30-34	160	23,000
35-39	155	11,000
40-44	125	2,000
45-49	87	125
Total		

The total population of Assam in 1988 was 4,000.5 thousands.

Determine (a) the crude birth rate, (b) the general fertility rate, (C) the age specific rates and (d) the total fertility rate for 1988.

4.9 FURTHER READINGS

1. Goon A.N., Gupta M.K. & Das Gupta B Applied Statistics Vol. II The World Press Pvt. Ltd., Kolkata.
2. Gupta S.C. and Kapoor V. K., Applied Statistics, S Chand, New Delhi.
3. Ramkumar, Demography

Uttar Pradesh Rajarshi Tandon
Open University

MFN -107

Biostatistics

BLOCK

3

TESTS OF SIGNIFICANCE

Course Design Committee

Dr. (Prof.) Ashutosh Gupta — **Director**
School of Science, UPRTOU Prayagraj

Prof. Umesh Nath Tripathi — **Member**
Department of Chemistry
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. S.I. Rizvi — **Member**
Department of Biochemistry
University of Allahabad, Prayagraj

Prof. Dinesh Yadav — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Prof. Sharad Kumar Mishra — **Member**
Department of Biotechnology
Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur

Dr. Ravindra Pratap Singh — **Member**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Dr. Dharmveer Singh — **Member/Secretary**
Academic Consultant (Biochemistry)
School of Sciences, UPRTOU Prayagraj

Course Preparation Committee

Dr. Shruti — **Writer**
Sr. Assistant Professor, School of Sciences
U. P. Rajarshi Tandon Open University, Prayagraj

Prof. G. S. Pandey — **Editor**
Department of Statistics, University of Allahabad,
Prayagraj

Dr. Dharmveer Singh — **Course Coordinator**
Academic Consultant (Biochemistry)
School of Science, UPRTOU, Prayagraj

ISBN - 978-93-94487-67-3

Printed and Published by Vinay Kumar, Registrar, Uttar Pradesh Rajarshi Tandon Open University, Prayagraj - 2024.

Printed by - K. C. Printing & Allied Works, Panchwati, Mathura -281003

BLOCK INTRODUCTION

The present SLM on ***Bio Statistics*** consists of three Blocks. *Block - 1 – Research Methodology and Statistical Methods* has two units; *Block - 2 – Probability - Distributions and Demography* has two units; and at the last *Block – 3 Tests of Significance and Analysis of Variance* has two units.

The ***Block-1– Research Methodology and Statistical Methods*** consists two units. The *first unit* of this block; named *Research Methods and Sampling Procedures*; describes the meaning and types of research, significance of research. It tells about the research problem and its selection with *Sampling Theory*; which discuss about the sampling, different types of sampling designs, simple random sampling, stratified sampling and cluster sampling with their applications.. The *second unit* of this block; named *Statistical Tools*; discusses about the measures of central tendency, measures of dispersion, measures of asymmetry, correlation and regression analysis, association of attributes and 3-sigma limits.

The ***Block-2– Probability- Distributions Theory and Demography*** is the second block having two units. The *first unit* of this block; named *Probability and Distribution Theory*; gives the Basic concepts of probability, definitions of probability, additive and multiplicative law of probability, conditional probability, Bayes' theorem, random variable and its types, probability mass function and probability density functions. In *Probability Distributions*, the concept of probability distribution, discrete and continuous probability distributions namely Binomial Distribution, Poisson Distribution, Geometric Distribution, Normal Distribution, Exponential Distribution have been also discussed in this unit along with their properties, applications and importance.. The *second unit* of this block; named *Demography*; gives knowledge about the vital statistics and demography, this also tells about the source of vital statistics and demographic data, rates, ratio, proportion, measures of fertility, measures of mortality, measures of morbidity and migration.

The ***Block-3– Tests of Significance and Analysis of Variance*** consists of two units. The *first unit* of this block; named *Testing of Hypothesis*; discuss about the hypothesis and its types, level of significance, critical region, p-value, types of errors, chi-square tests, t-tests and z-tests with their applications. The *second unit* of this block; named *Analysis of Variance*; discusses about the concept of analysis of variance and co-variance, basic principles of ANOVA and ANCOVA. (One Way, Two Way and Three Way Analysis).

Illustrations and examples on these topics have also been given.

At the end of every block/unit the summary, self assessment questions and further readings are given.

UNIT-5 TESTING OF HYPOTHESIS

Structure

5.1 INTRODUCTION

There are two major areas of statistical inference namely the estimation of parameter and the testing of hypotheses. Our present aim is to introduce to concepts involved in the development of general methods for testing of hypotheses. Some illustrations are taken from population having the common known distributions. In all the problems of statistical inference there is generalization of the results or conclusions of a sample(s) from the population to the population itself. The error is possible. We shall explain the two kinds of error in the context. In testing to hypotheses a decision is taken on the basis of a samples (s) whether to accept or to reject a specified value Ho: θ θ or a set of specified values Ho: $\theta \in \theta$ where $Q\ C\ \theta$ On the basis of the results of sample (s) from the population f(x; θ), $\theta \in \theta$ where θ may be vector and θ is the parameter space of θ, for example in exponential distribution f(x; θ)= $\theta\, e$ $, x$ $0, \theta$ 0; θ is a single parameter and but in normal density f(x; θ) $\frac{}{\sqrt{=}}\, e$ $\frac{}{\Gamma}$

Where ,

$$\infty \quad x \quad \infty, \sigma \quad 0, \quad \infty \quad \mu \quad \infty$$

Unknown parameter μ and σ are unknown the θ $, \sigma\, \mu$ and its parameter space is θ: (∞ μ ∞, σ 0).

In the present discussion the sample size n is considered fixed in the

advance, The case of sequential analysis, discussed by A. Wald, where are is used are not considered.

A pharmaceutical concern may be interested to find if a new drug is really effective for treatment of an element say cancer or in reducing blood pressure or inducing sleep; One may want whether a new foodstuff is really effective in increasing weight; or which of the two brands of particular product say, food stuff, fertilizers, etc. is more effective; such practical problems may be quoted, where the modern probability theory plays a vital role in decision making and the branch of statistic which helps us in arriving at a criterion for such decision known as testing of hypothesis. The theory of testing of hypothesis was first given a mathematical sound footings by J. Neyman and E.S. Pearson through series of papers. They dealt with the statistical techniques to arrive at decision in certain situations where there is an element of doubt, on the basis of a sample whose size is fixed in advance. There is another technique known as sequential testing pro-founded by Abraham Wald where the sample size is not fixed I advance but is regarded as a random variables.

5.2 OBJECTIVES

After studying this unit you shall be able to:

- Understand testing of hypothesis.
- Decide a null hypothesis and alternative hypothesis.
- Understand the meaning of level of significance, size of a test and power of a test.
- We shall define some terms associated with testing of hypothesis.

5.3 HYPOTHESES AND ITS TYPE

A statistical hypothesis is some assumption or statement about a population, or probability distribution characterizing the given population. It is frequently denoted by H.

A hypothesis is not accepted without being supported by evidence from the population. A hypothesis needs to be verified and is therefore, put to test; and based on the evidences provided by a random sample (which are set of independent observations) from the population a decisions is taken to accept or reject it. For example if r.v. X ~N (μ, 25) then the statement that the mean of the population is greater than 20, is a statement about the population mean with known variance σ = 25 and therefore is a hypothesis. We write H: μ >20.

Simple and Composite Hypotheses- A hypothesis is known as *simple hypothesis* if it completely specifies the population; otherwise it is known as a *composite hypothesis.*

For example, in sampling from a normal population N (μ, σ), the hypothesis.

(a) $Ho: \mu \quad \mu, \quad \sigma \quad \sigma o$

is a simple hypothesis. It specifies values to both parameter μ and σ ;and therefore, it completely specifies the distribution, On the other hand each of the following hypotheses is composite hypothesis:

(ii) $Ho: \mu$ μ (No statement about σ)

(iii) H: σ σo μ *is not specified*

(i) H: μ μ , σ σo

(ii) $H: \mu$ μ , σ σo

(iii) $H: \mu$ μ , σ σo

and so on

Null hypothesis and Alternative Hypotheses

a statistical hypothesis is said to be *Null hypothesis* which is put to test for possible rejection under the assumption that it is true; it is denoted by Ho.

For example in sampling from normal population N (μ, σ) the hypothesis $Ho: \mu$ μ is a null hypothesis if it is to be tested. It is said to be a null hypothesis since it states that there is no difference between $Ho: \mu$ *and* μo.

It is very important to state the alternative hypothesis H_1 explicitly in respect to any null hypothesis Ho because the acceptance or rejection of Ho is meaningful only if it is being tested against the rival hypothesis H_1.

The concept of simple and composite hypothesis applies also to alternative hypothesis. For example in comparing the mean effect on the yield of soyabean of two fertilizer say A and B, we may formulate the null and alternative hypothesis as Ho: μA μB against H_1: μA μB. Ho is a simple hypothesis and H_1 is a composite hypothesis.

If we want to test the null hypothesis Ho that the population N (μ, σ) has specified mean μ let be σ known then

The null hypothesis $Ho: \mu$ μ, σ 0

and alternative hypothesis $Ho: \mu$ μ, σ 0

5.4 CRITICAL (REJECTION) REGION

Mathematically,

Let the population be X~ f (x; θ) $\theta \in Q$ where Q is the parameter space of the parameter θ . Let $\underline{x}$: $x x \ldots\ldots x$ be n independent sample observations corresponding to a random sample $\underline{X}$: $X X \ldots\ldots X$ of size n from the population. The n-dimensional space S which is the aggregate of all sample points $\underline{x}$: $X X \ldots\ldots X$ is called *sample space and* is denoted by S.

The test for a hypothesis divides the whole sample S into two disjoint (mutually exclusive) regions; one region A for acceptance of hypothesis H and another region R (or C) for rejection of hypothesis H.

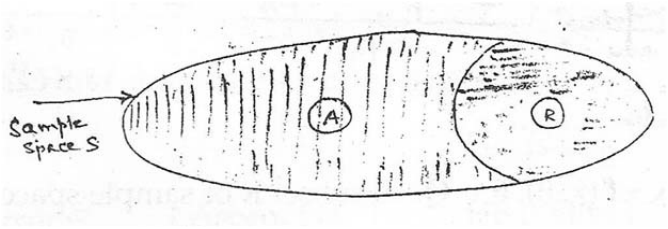


Figure 3.1 Sample Space, Acceptance (A) and Rejection (R)(or critical) region

Thus the test for hypothesis H is;

Rejected Ho if $x_1 x_2 \ldots \ldots x_n \in R$,

Accepted Ho if $x_1 x_2 \ldots \ldots x_n \in A$

A statistic associated with the test is called a test statistic.

Let X~ f (x; θ) $\theta \in Q$. A subset R of sample space S, such that if R then Ho is rejected (with probability 1) is called the *critical region (or rejection region)* C of the test, where

$$C = \{x \in S : H \text{ is rejected if } \underline{x} \in R\}$$

The complementary set A or R is said to *acceptance region* of the test.

5.5 TYPES OF ERROR AND LEVEL OF SIGNIFICANCE

The decision of the test for hypothesis H is taken on the basis of the information of a sample from the population. As such there is an element of risk – the risk of taking wrong decisions. In any test procedure, there are four possible mutually exclusive and exhaustive decisions:

(i) Reject Ho when actually Ho is not true (false)
(ii) Accept Ho when it is true
(iii) Reject Ho when it is true
(iv) Accept Ho when it is false

The decisions in (i) and (ii) are correct while the decisions (iii) and (iv) are wrong decisions. These decisions may be expressed in the following dichotomous table.

True state in The nature	Decision		
		H_1 True	H_1 False
	Ho True	Wrong (Type I error)	Correct
	Ho False (H_1 True)	Correct	Wrong (Type II error)

Thus in testing hypothesis may lead to following two kinds of errors.

An error of type I is made if the null hypothesis Ho is rejected when Ho is true; and the error of Type II is made if the null hypothesis Ho is accepted when Ho is false.

Type I Error = [Reject Ho | Ho is true]

Type II Error = [Accept Ho | Ho is false]

= [Accept Ho | Ho is true]

= [Reject H_1 | Ho is true]

The *probabilities of type I and Type II errors* are denoted by α *and* β respectively.

Definition- The *size of a Type I error is the probability of type I error* α *similarly, the size of a type II* error is the *probability of type II error* β

Thus, ***Level of significance-***

α = Probability of Type I error

= Prob. [Reject Ho |Ho]

The *level of significance* is the maximum of probability of the type I error with which one is prepared to reject Ho when Ho is true. It is also called the size of critical region.

5.6 TEST OF SIGNIFICANCE

Suppose that the problem is to test the hypothesis that the mean μ of the normal population $N(\mu, \sigma)$ with known variance σ is different from μ. As explained above the null hypothesis Ho and the alternative hypothesis H_1 will be set up as follows:

$$H : \mu \quad \mu \ against\ alternative\ H \quad \mu \quad \mu$$

A test of significance for hypothesis Ho: is a procedure to assess the difference between the sample statistic and the value of parameter given by Ho or differences between two independent statistic to be significant or to reject or accept Ho at the given level of significance α.

The procedure to be adopted for test of significance is outlined below-

- Propose the null hypothesis Ho and alternative hypothesis H_1.
- Fix a level of significance α for the test and a sample size n.
- Then choose a statistic T(x) whose sampling distribution is known under H_o.
- Keeping the value of α in mind decide upon those values of the test statistic (i.e. rejection region) that lead to its acceptance. In other words, define the test for H_o Vs. H_1 at level α.
- Now draw a random sample of size in from the population and compute the value of the test statistic.

- Finally on the basis of the value of the test statistic take the decision to accept or reject Ho.

5.7 TEST OF SIGNIFICANCE BASED ON CHI-SQUARE TEST

(a) To test the signification of variance from a normal population.

Let $X_1, X_2, \ldots\ldots X_n$ be random sample of size n taken from N (μ, σ) and w wish to test Ho: $\sigma \quad \sigma$.

If μ is known then H_0

$$\frac{\sum X \quad \mu}{\sigma}$$

Follows Chi-square distribution with n degree of freedom.

If μ is not known then under H_o

$$\frac{\sum X \quad X}{\sigma}$$

Follows Chi-square distribution with (n-1) degree of freedom;

$X \quad -\sum X$ Is the sample mean.

(b) To Test Goodness of Fit.

Here we test the null hypothesis that data follow a particular probability distribution (like Binomial, Poisson, Normal etc.).

Let O_1, O_2, be the observe frequencies and $e_1, e_2,\ldots$ are the expected frequencies corresponding to these observed frequencies then

$$\frac{0 \quad e}{e} \qquad \frac{o}{e} \quad N \qquad where\ N \quad O \quad e$$

follows a chi-square distribution with certain say v degrees of freedom. If expected frequencies are less than 5, then pooling is done.

Example- Five dice were thrown 192 times and the number of times 4, 5 or 6 were as follows:

No. of dice throwing 4,5,6	5	4	3	2	1	0	Total
Observed frequency: O_i	6	46	70	48	20	2	192

Calculation the value of Chi-Square on the hypothesis that dice were unbiased and hence test whether the data are consistent with the hypothesis.

Solution- Probability of throwing 4, 5, 6 is 3/6= ½= p

Therefore from binomial distribution the theoretical frequencies of getting 5,4,3,2,1,0 successes with 5 dice re respectively the successive terms of :

$N (p+q)^5 = 192(1/2+1/2)^5$

Which are as follows: 6, 30, 60, 30, 66, 36 respectively

Table showing observed and expected frequency after pooling the frequency which is less than 5.

No. of dice throwing 4,5,6	5	4	3	2	1or 0	Total
Observed frequency: O_i	6	46	70	48	22	192
Expected frequency: ei	6	30	60	60	36	192

$So\ \chi$ $\frac{0 \quad e}{e}$

$\frac{6 \quad 6}{6}$ $\frac{46 \quad 30}{30}$ $\frac{70 \quad 60}{60}$ $\frac{48 \quad 40}{60}$ $\frac{22 \quad 36}{36}$

0 8.53 1.66 2.4 5.44 18.03

There are n= 6 cells and k =1 cell, i.e., "0" is polled with cell "1", therefore the degree of freedom of is

V=n-1-k, = 6-1-1=4.

The tabulated value of Chi-square (χ) on 4 degrees of freedom and at 5 % level of significance is 9.488. Since the calculated value of χ is greater than the tabulated value hence null hypothesis be rejected. Therefore the observed frequency distribution is not consistent with the hypothesis.

(c) Testing of independence or Association between two (attributes) (characters):

If a character (factor, attribute) A is classified into A_1, A_2, A_i, …..A_r classes and second character (factor/ attribute) B into B_1, B_2,….B_i….B_c classes and if O_{ij} is the observed frequency due to Ai class of A and Bi class B which are shown in the following table:

Character B

Character A		B_1	B_2	B_j	B_c	R_1
	A_1	O_{11}	O_{12}	O_{2j}	O_{1c}	R_1
	A_2	O_{21}	O_{22}	O_{2j}	O_{2c}	R_2
	A_i	O_{i1}	O_{i2}	O_{ij}	O_{ic}	R_i

	A_r	O_{r1}	O_{r2}	O_{tkj}	O_{rc}	R_r
	Total	C_1	C_2	C_j	C_{rc}	N

Where,

$$R \qquad C \qquad N\ grand\ total$$

Then this table is called r x c contingency table.

We are interested in testing the null hypothesis.

H_0: Character (attributes) A and B are independence, i.e., there is no associated between two character A and B.

Let eij denotes the expected frequency due to i^{th} class of A and j-th class of character B, i= 1, 2,……r, j= 1,2,……,c, Then

$$e \qquad \frac{R \quad C}{N} \qquad N\frac{R}{N} \quad \frac{C}{N}$$

Thus, the expected frequency of any cell is equal to the product of the class totals of the two classes to which the cell belongs divided by the total number of observations.

Hence to test the above null hypothesis, we use Chi-square as given by:

$$\frac{0 \quad e}{e} \qquad \frac{O}{e} \quad N$$

It is a Chi statistic with (r-1) (c-1) degrees of freedom.

The calculated value of is compared against the table value of χ on (r-1) (c-1) degrees of freedom and 5% probability level. If calculated value of χ is greater than its table value then the null hypothesis H_0 is rejected, otherwise H_0 will be accepted.

Rejecting the null hypothesis means that there is association between two factors (attributes/ character).

Example- From a village 200 persons were randomly selected and data about their income and education achievement were recorded, which are given in the following table.

Education

		High	Medium	Low	Total
Income	High	60	20	20	R_1=100

	Low	20	20	60	R_2=100
Total		e_1=80	e_2=40	e_3=80	N= 200

Test whether education depends upon income.

Null Hypothesis

Against the alternative

H_1; There is association between education and income.

Expected frequencies

e ——— 40, e ——— 20, e ——— 40, e ———

40, e ——— 20, e ——— 40

Table of expected frequencies:

Education

		High	Medium	Low	Total
Income	High	40	20	40	100
	Low	40	20	40	100
Total		80	40	80	200

$$\chi \quad \frac{0 \quad e}{e}$$

$$\frac{60 \quad 40}{40} \quad \frac{20 \quad 20}{20} \quad \frac{20 \quad 40}{40} \quad \frac{20 \quad 40}{40} \quad \frac{20 \quad 20}{20}$$

$$\frac{60 \quad 40}{40}$$

$$\frac{20 \quad 20}{40} \quad 200 \quad \frac{20 \quad 20}{40} \quad \frac{20 \quad 20}{40} \quad 0 \quad \frac{20 \quad 20}{40}$$

$$10 \quad 0 \quad 10 \quad 10 \quad 0 \quad 10 \quad 40$$

d.f.= (r-1) (c-1) = (2-1) (3-1) = 1 2 = 2

The tabulated value of χ and 2 d.f. and at 5% level of significance is 5.991.

Since the calculated value of χ is greater than the table value of χ on 2 degrees of freedom and at 5% probability level so our null hypothesis will be rejected. Therefore, it can be concluded from the above data that there between education and income.

5.8 TESTS OF SIGNIFICANCE BASED ON T-TEST

t- test is based on t- distribution and used for testing; the significance of mean from a normal population, the significant different between two population means of normal population, the significance of correlation coefficient from a bivariate normal population and for the significance of regression coefficient.

(i) Testing the significance of mean from a normal population.

Suppose x_1, x_2 , x_3……..x_n is a random sample of size n from a normal population with mean μ and variance σ (σ hot known) and one wishes to test. Ho:

Under Ho: μ μ

$$t \quad \frac{\bar{x} \quad \mu \quad \sqrt{n}}{S} \; follows$$

t distribution with (n-1) degrees of freedom. Here, $\bar{x}$ is the sample mean and

$$S \quad \frac{1}{n \quad 1} \quad x \quad \bar{x}$$

Where S is an unbiased estimate of σ

The calculated of |t| is compared against the tabulated value of t. If the calculated value of |t| is greater than the table value of t on (n-1) degrees of freedom and at $\propto$ % probability level, the above null hypothesis will be rejected at $\propto$–level of significance otherwise, H0 will be accepted.

Example- Ten rice plants were randomly selected from a small research plot having 100 plants. The height of these plants was recorded to study the effect of a bio-fertilizer on the growth behavior of plants which are given below-

Height in cm: 80,76,78,84,82,83,77,80,81,79

In the light of the above data, test whether the average height of plants in the population in 82.5 cm.

X height in cm	$X \quad X$	$X \quad X$	Ho: μ 82.5
80	0	0	
76	-4	16	$S \quad \frac{\sum \quad x \quad \bar{x}}{n \quad 1}$
78	-2	4	
84	+4	16	= 60/9 = 6.666
82	+2	4	
83	+3	9	S= $\sqrt{6.666}$
77	-3	9	

80	0	0	S= 2.58= 2.58
81	+1	1	
79	-1	1	
Total 800 = $\sum X$	-	60 =$\sum X \quad X$	

$X \quad \frac{\Sigma}{} \quad — \quad 80 \quad t \quad \frac{-}{\sqrt{}} \quad \frac{\sqrt{}}{.} \quad \frac{.}{.} \quad . \quad 3.07$

$or\ |t| \quad 3.07$

Table value of t on 9 d.f. and at 5% level of significance is 2.262.

Since the calculated value of |t| is greater than the table value of t on 9 degrees of freedom and at 5% probability level, so our null hypothesis Ho will be rejected. Therefore it can be concluded from the given data that population mean is significantly different from = 82.5 cm., in other words the average height of plants in the population cannot be regarded as 82.5 cm from which a random sample of 10 plants have been selected with sample mean 80 cm.

(ii) Testing the significant different between two population means

Suppose $x_1, x_2, x_3, \ldots x_{n1}$., is a random sample from 1st Normal population with mean μ and variance $\sigma 1$ and another independent random sample $y_1, y_2, \ldots\ldots y_{n2}$ from 2nd normal population with mean and variance. It is further assumed that $\sigma 1 \quad \sigma 2 \quad \sigma$, (say) unknown

We wish to test the null hypothesis H_0= : $\mu \quad \mu$

Under Ho

$$\frac{\bar{x} \quad y}{S \ \frac{1}{n} \quad \frac{1}{n}}$$

Follows t-distribution with (n_1-n_2) degrees of freedom. Where

$$S \quad \frac{\sum \quad x \quad \bar{x} \quad \sum \quad y \quad y}{n \quad n \quad 2} \quad \frac{n \quad 1\ S \quad n \quad 1\ S}{n \quad n \quad 2}$$

Is an unbiased estimated of σ , The calculated value of |t| is compared against tabulated value of t on ($n \quad n \quad 2$) d.f. and at α% level of significance. If cal. |t|> t ,$n \quad n \quad 2$ then the null hypothesis Ho is rejected otherwise Ho is accepted. The above procedure is called as ***two sample t-test***.

Example- Ten red plants were randomly selected from 1st plot and 8 yellow plants were randomly selected from second rose plot. The height of these selected were separately recorded and are given below in cm.

Heights of red rose plants in cm = xi	$X \quad X$	$X \quad X$	Heights of yellow rose plants in	$y \quad y$	$y \quad y$

			cm = yi		
60	0	0	62	0	0
64	4	16	60	-2	4
61	1	1	63	+1	-
56	-4	10	64	+2	4
59	-1	1	61	-1	1
62	2	4	63	+1	1
58	-2	4	63	+1	1
60	0	0	61	-1	1
63	3	9			
57	-3	9			
$\sum X$ =600		$\sum X$ X =60	$\sum y$ =496		$\sum X$ X =12

Discuss the suggestion whether there is significant different between the mean height of red and yellow row plants.

Ho: There is no different between the mean heights of red and yellow rose plants.

$$\bar{x} \quad \frac{\sum x}{n} \quad \frac{600}{10} \quad 60 \qquad y \quad \frac{\sum y}{n} \quad \frac{496}{8} \quad 62$$

$$S \quad \frac{\Sigma \quad - \quad \Sigma}{\qquad\qquad\qquad} \quad \frac{\qquad}{\qquad} \quad \_ \quad 4.5$$

$$S \quad 4.5 \quad S \quad \sqrt{4.5} \quad 2.12$$

Therefore,

$$t \quad \frac{\bar{x} \quad y}{S \sqrt{\frac{1}{n} \quad \frac{1}{n}}} \quad \frac{60 \quad 62}{2.12 \sqrt{\frac{1}{8} \quad \frac{1}{10}}}$$

$$t \quad \frac{2}{2.12 \quad 3.5} \quad \frac{2}{0.742} \quad 2.7 \quad |t| \quad 2.7$$

The calculated value of |t| is greater than the table value of t on 16 d.f. and at 5% probability level, so our null hypothesis is rejected. Therefore it can be concluded that our null hypothesis is rejected, hence there is significant different between the mean heights or red and yellow rose plants.

(iii) Paired t-test

Let (x_1,y_1), (x_2,y_2), …… (x_n, y_n) be a random sample of size n drawn from a bivariate normal population $\mu\ \mu\ \sigma\ ,\sigma\ ,\rho$ we wish to test

H_o: μ μ

In this case, under H_o:

$$\frac{\bar{d}\sqrt{n}}{S}$$

Follows t-distribution with (n-1) degrees of freedom.

Where

d x y, $\bar{d}$ $-\sum d$, S $-\sum$ d $\bar{d}$

Example of paired observations-

If we want to compare the effects of two drugs D_1 and D_2 for the same disease, then first of all drug D_1 should be administered to a set of certain patients (i= 1,2,3, n) and its effect should be recorded. After a reasonable interval of time, the drug D_2 should be administered to the same set of patients and its effect should be recorded. The observations so obtained are said to be paired.

Example- The scores of 10 cadets before and after training gave below:

Cadet No.	1	2	3	4	5	6	7	8	9	10
Score before training x_i	8	3	2	4	6	8	5	8	7	9
Score after training y_i	7	5	7	5	3	9	5	10	6	10

Based on the above data can you say whether training is effective in improving the performance of cadets.

Solution-Since observation under x and y are paired, so to test the null hypothesis.

H_o: Training is not effective in improving the performance of cadets i.e.,

H_o: μ μ against the alternative H_1: μ μ we use the paired t-test and test statistic.

$$t \quad \frac{\bar{d}}{S\ /\sqrt{n}}$$

d x y	-1	2	5	1	-2	1	0	2	-1	1	$\sum d$ =7
di	1	4	25	1	4	1	0	4	1	1	$\sum di$ = 47

$$\bar{d} \quad \frac{\sum d}{n} \quad \frac{7}{10} \quad 0.7$$

$$S \quad \frac{\Sigma}{} \quad \frac{\Sigma}{} \quad \frac{}{} \quad \frac{.}{} \quad .$$

$$\frac{4.21}{9} \quad 2.15$$

Here,

$$S \quad \sqrt{4.65} \quad 2.15$$

We have

$$t \quad \frac{\bar{d}\sqrt{n}}{S} \quad \frac{0.7 \quad \sqrt{10}}{2.15} \quad \frac{0.7 \quad 3.16}{2.15} \quad 1.28$$

Table value of t (5%) on 9 d.f. is 2.262

The calculated value of t is less than the table value of t on 9 degrees of freedom and at 5% probability level. Therefore, null hypothesis may be accepted and it can be conducted that training is not very effective in improving the performance of cadets.

iii) Test of significance of correlation coefficient

Suppose a pair of random sample (x_1,y_1); (x_2,y_2)….(x_n,y_n) is drawn from a bivariate normal population with population correlation p.

Let the sample correlation between x and y be r

Then to test the null hypothesis:

H_o: p=0 (i.e. population correlation effective is zero) against the alternative H_1: p 0.

Under H_o; $\frac{\sqrt{}}{\sqrt{}}$ follows a t-distribution with (n-2) degrees of freedom.

Example- A random sample of size 18 form a bivariate normal population gave a correlation coefficient 0.6. Does it indicate the existence of correlation in the population?

H_o: p=0 Here, we wish to test H_o: p=0 Vs H_1: p 0.

The value of t=$\frac{\sqrt{}}{\sqrt{}}$ is

$$t \quad \frac{r\sqrt{n \quad 2}}{\sqrt{1 \quad r}} \quad \frac{0.6\sqrt{18 \quad 2}}{\sqrt{1 \quad .36}} \quad \frac{0.6 \quad 4}{.80} \quad \frac{2.4}{.8} \quad 3.0$$

The tabulated value of t on 16 degrees of freedom at 5% level of significance is 2.12.

Since the calculated value of t is greater than the table value of t on 16 d.f. and at 5% level of significance so our null hypothesis Ho is rejected. Therefore, it can be

concluded that the sample correlation coefficient r=0.6 is significant and it indicates the existence of correlation in the population.

(iv) To test the significance of Regression Coefficient:

Suppose $(x_1,y_1); (x_2,y_2)....(x_n,y_n)$ is a random sample from a bivariate normal population with regression coefficient β, of y and x

We know that the regression equation of y on x from the sample is: $y \quad y \quad b \quad x \quad \bar{x}$.

$\bar{x}, y$ are sample means and b is the sample regression coefficient of y or x.

So that the estimated value of y corresponding to given xi is

$$Y \quad y \quad b \quad X \quad X$$

We wish to test the null hypothesis:

H_o: $\beta \quad \beta$ where β is known

Under Ho:

$$\frac{b \quad \beta \quad \sqrt{n \quad 2} \sum x \quad \bar{x}}{\sum \quad y \quad y}$$

Follows t- distribution with (n-2) defines of freedom.

5.9 TEST OF SIGNIFICANCE BASED ON Z-TEST

Testing significance of mean- Let x_1, x_2, x_3,……..x be random sample of size n taken from N(μ, σ). The sample mean $\bar{x} \quad -\sum x$ follows normal distribution with mean μ and variance $\sigma \ /n$. For large samples, (i.e. ***where sample size is more than 30***), it is true even if population is not normal. Thus,

$$Z \quad \frac{\bar{x} \quad \mu \sqrt{n}}{\sigma}$$

be have large samples. However, if the parent population is normal then the result is true even for small samples.

With an increased sample size, the sample variance $S \quad -\sum x \quad \bar{x}$ can safely be taken as an approximation to population variance σ (if not known). Thus without any significant error approximation in large samples we have,

$$Z \quad \frac{\bar{x} \quad \mu \sqrt{n}}{s}$$

So if one wishes to test H_0: $\mu \quad \mu$ in case of large samples with unknown population variance one may use the test statistic

$$Z \quad \frac{\bar{x} \quad \mu \quad \sqrt{n}}{s}$$

For known variance one uses

$$Z = \frac{\bar{x} - \mu}{s}\sqrt{n}$$

To reject or accept the above null hypothesis, the calculated value of Z is compared with 1.96 which is table value of a standard normal variate at 5% probability level for two tail test and 1.645 for one sided test.

If the calculated value of | Z | is greater than 1.96, then we reject our null hypothesis Ho and otherwise it is accepted.

Example- A random sample of 400 male students is found to have a mean height of 168 cm. Can it be reasonable regarded as a sample from a population with mean height = 167.8cm. and standard deviation 3.25cm?

Here $\bar{x} = 168$, $\mu = 167.8$cm, $\sigma = 3.25$, $n = 400$.

So

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{168 - 167.8}{\frac{3.25}{\sqrt{400}}} = \frac{0.2 - 20}{3.25} = \frac{4.00}{3.25} = 123$$

The calculated value of is smaller than the table value of Z = 1.96 at 5% probability level, so our null hypothesis will be accepted. Hence there is no significant difference between sample mean and population mean. Therefore it can be reasonably regarded as a random sample from a population with mean 167.8cm and $\sigma = 3.25$

Testing Equality of Means- Suppose $x_1, x_2, \ldots\ldots\ldots x_n$ is an random sample from a normal population with μ as mean and variance σ and another random sample $y_1, y_2, \ldots y_{n2}$ from second normal population with mean μ and σ . It is assumed that and are large two random samples are independent. Then we wish to test the null hypothesis. n_1 and n_2 are large and two random samples are independent. Then we wish to test the null hypothesis.

Ho: μ μ

Now we know that

$\bar{x} \sim N(\mu, \_\_)$ and $y \sim N(\mu, \_\_)$

Hence,

$$Z = \frac{\bar{x} - y - (\mu - \mu)}{\sqrt{\frac{\sigma}{n} - \frac{\sigma}{n}}} \text{ is a standard normal variate } S.N.V.$$

So, if Ho: μ μ is true then

$$Z \quad \frac{\bar{x} \quad y}{\sqrt{\frac{\sigma}{n} \quad \frac{\sigma}{n}}} \; is\ a\ standard\ normal\ variate \qquad S.N.V.$$

Hence if calculated value of | Ƶ | is greater than 1.96 which is table value of Ƶ at 5% probability level, then our null hypothesis Ho will be rejected, otherwise accepted. The table value of Ƶ at 1% level of significance is 2.58.

Example- A random sample of 150 village was taken from district A and the average population per village of was found to be 440 and standard devotion 32. Another random population of 250 villages from the same district gave an averaged population 480 per village with standard

Here n_1= 150 $\bar{x}$ = 440 s_1= 32

n_1= 250 y = 480 s_2= 56

We want to test the null hypothesis

Ho: μ μ

Under Ho,

$$Z \quad \frac{\bar{x} \quad y}{\sqrt{\frac{s}{n} \quad \frac{s}{n}}} \qquad \frac{440 \quad 480}{\sqrt{\frac{32}{150} \quad \frac{56}{250}}}$$

$$\frac{40}{\sqrt{6.83 \quad 12.542}} \qquad \frac{40}{\sqrt{19.372}} \qquad \frac{40}{4.42} \qquad 9.09$$

Hence | Ƶ |= 9.09

Since the calculated value of | Ƶ | is greater than the table value of Ƶ = 2.58 at 1% level of significance. Therefore the different between the means is highly significant.

Testing Significance of Proportion- If a random sample of size n is drawn from a population with population proportion P. then we wish to test the null hypothesis

Ho: P= P_o where P_o is a particular specified value of P

Standard error of sample proportion is

$$SE \quad p \qquad \sqrt{\frac{PQ}{n}} \; where\ Q \quad 1 \quad P$$

and n is large

Then to test the above null hypothesis, under Ho,

$$Z \quad \frac{p \quad p}{\frac{P\ Q}{n}} \; is\ a\ Standard\ Normal\ Variate \quad SNV \; ; \; Q \qquad 1 \quad P$$

if calculated value of | Ƶ | is greater than the table value of Ƶ at 5% level of significance, our null hypothesis will be rejected, otherwise accepted. (the table value of Ƶ = 1.96 at 5% level for two sided test and the table value of Ƶ = 1.645 for one sided test)

Example- In a sample of 400 burners, there were 12 burners whose internal diameters were not within tolerance. Is this sufficient evidence for concluding that the manufacturing process is turning out more than 2% defective burners?

Here P= .02, Q= 1-P= 0.98 and p= 12/400= .03

The null hypothesis is:

Ho: P= 0.02

To test the above null hypothesis we use:

$$Z \quad \frac{p \quad p}{\frac{P\ Q}{n}} \quad \frac{.03 \quad .02}{\frac{.02 \quad .98}{400}}$$

$$\frac{0.01}{\sqrt{.000049}} \quad \frac{.01}{.007}$$

$$1.429$$

The calculated value of is less than the table value of = 1.645 (for one tail test) at 5% probability level, so our null hypothesis is accepted. Therefore, it can be concluded that the process is under control.

Testing Equality of Proportions- Let n_1 and n_2 be two large samples taken from two different population and we wish to test Ho: $P_1=P_2$, where P_1 and P_2 are population proportions of two quality characteristics. Suppose p_1 and p_2 be the corresponding sample proportions obtained from the two random samples drawn these populations. If n_1 and n_2 are sufficiently large, then under Ho:

$$Z \quad \frac{p \quad p}{\frac{p\ q}{n} \quad \frac{p\ q}{n}} \sim N \quad 0,1$$

When $q_1= 1-p_1$ and $q_2= 1-p_2$

However if n_1 and n_2 are moderately large with define

$$p \quad \frac{n\ p \quad n\ p}{n \quad n} \; and\ q \quad 1 \quad p.$$

In this case under Ho,

$$Z \quad \frac{p \quad p}{pq \ \frac{1}{n} \quad \frac{1}{n}} \sim N \quad 0,1$$

Example- A machine produced 20 defective articles in a batch of 400. After overhauling it produced 10 defectives in a batch of 300. Has the machine improved?

Ho: there is no difference in the improvement of the machine before and after overhauling.

Or Ho: p_1-p_2= 0

P_1= 20/400= .05 so q_1 = 1-p_1= 1-.05= .95

P_2 = proportion of defective articles after overhauling = 10/300= .033

So q_2= 1-p_2= 1-.033= 0.97

$$SE \ p \quad p \qquad \frac{p \ q}{n} \quad \frac{p \ q}{n}$$

$$\frac{.05 \quad .95}{400} \quad \frac{.033 \quad .967}{300}$$

$$\sqrt{0.00023} \quad 0.015$$

Then

$$Z \quad \frac{p \quad p}{SE \quad p \quad p} \quad \frac{.05 \quad .033}{.015} \quad \frac{017}{.015} \quad 1.134$$

The calculated value of | Ƶ | is smaller than the table value of Ƶ at 5% level of significance, so our null hypothesis Ho is accepted. Therefore, it can be concluded that the machine has not improved after overhauling

Testing significance of standard derivation- The variance of a sample standard deviation say's is — if a large sample of size n is drawn from a normal population of variance. It is to be noted that if parent population is not normal then this formula is not to be relied upon. Thus, for normal parent population to test Ho: $\sigma \quad \sigma$

Under Ho:

$$Z \quad \frac{s \quad \sigma}{\frac{\sigma}{2n}} \ is \ a \ S.N.V.$$

Testing Equality of Standard Deviations- Let s_1 and s_2 be the sample standard deviation of the two large samples of sizes n_1 and n_2 taken from two normal population with variance $\sigma \ and \sigma$ respectively. For testing Ho: $\sigma \quad \sigma$ under Ho, we have

$$Z = \frac{s - s}{\sqrt{\frac{\sigma}{2n} + \frac{\sigma}{2n}}} \qquad \phi\sigma \text{ known}$$

Or

$$Z = \frac{s - s}{\sqrt{\frac{s}{2n} + \frac{s}{2n}}} \qquad \phi\sigma \text{ not known}$$

Or

$$Z = \frac{s - s}{s\sqrt{\frac{1}{n} + \frac{1}{n}}}$$

Where

$$S = \frac{1}{n + n}(n\,s + n\,s)$$

as standard Normal Variate (S.N.V.).

Example- Random samples of sizes 300 and 270 taken from two normal population with variance σ $and\sigma$ respectively gave sample standard deviations as 240 and 202.

Test the hypothesis Ho: σ σ at 1% level of significance.

Solution- Here Ho: σ σ Against H_1: σ σ

Under Ho,

Z [illegible] $-$ 3

For two sided test the tabulated value of Z at 1% level of significance is 2.58.

As calculated value falls in the critical region, Ho may be rejected at 1% level of significance and we may conclude that the difference between the standard deviations is significant at 1% level of significance.

5.10 SUMMARY

In this unit an attempt is made to explain the basic concepts related to the testing of hypotheses.

5.11 CHECK YOUR PROGRESS

1. In a rat feeding experiment the following results were obtained gain in weight in gm.

High Protein	13	14	10	11	12	14	10	8	11	12	9	12
Low Protein	7	11	10	8	10	12	9					

Find if there is any evidence of superiority of one diet over the other.

Given t (5%) on 17 d.f. is 2.11

Ans. t= 1.77

2. A random sample of 900 members is found to have a men of 3.4cms. Could it be regarded as a sample from a large population with mean 3.25cms? and s.d. 2.61 cm. at 5% level of significance/

3. The mean of sample of size 25 from a normal population with mean μ and s.d. 4 is found to be 15. Do you accept or reject Ho: $\mu = 20$ at the 10% level of significance?

5.12 FURTHER READINGS

1. A.M. Mood, F. Ar. Graybill & D.C. Boes. Introduction to the theory of Statistics, III. Editions Pub: Mac.Graw Hill.

2. Rohtagi V.K. (1984): An Introduction to Probability theory and Mathematical Statistics chapter VIII, IX & X Pub; John Wiley & Sons, New York.

3. Goon A.N., Gupta M.K. & Das Gupta B (1987) Fundamentals of Statistics Vol. I The World Press Pvt. Ltd., Kolkata.

4. Kapoor V.K. & S.C. Saxena: Fundamentals of Mathematical Statistics, Chapter Seventeen, Pub: S. Chand.

UNIT-6 ANALYSIS OF VARIANCE

Structure

6.1 INTRODUCTION

Frequently biologists select their samples from numerous different populations. The differences among more than two population means cannot be tested by the methods which described earlier. Analysis of variance (ANOVA) is used for determine whether or not the means of more than two populations are equal. Analysis of variance is large area of application of statistics developed by Prof. R.A. Fisher. The term "analysis of variance" is used because the total variability in the set of data can be broken into the sum of variability among the sample means and the variability within the samples. This procedure is based on the question- is there significantly more variation among the group means than there is within the groups? The pooled variation within groups is used as a standard of comparison, because it measures the inherent observational variability in the data. The difference in means should be large relative to this inherent variability if it is to be meaningful.

6.2 OBJECTIVES

After going through this unit you should be able to

- Understand the role of analysis of variance.
- Know and apply the basic principles on experimental units.

6.3 ANALYSIS OF VARIANCE

Once the statistical problem is formulated, the next step is to perform experiment for collecting relevant information regarding the statistical hypothesis

by an appropriate method which forms the basis for drawing valid inferences in the best possible manner. After the observations are obtained they are statistically analyzed using proper techniques so as to get relevant information regarding the objective of the experiment. Usually the objective or hypotheses of the experiments are to make comparison among the effects of several treatments. If the number of treatments is more than two, then analysis of variance technique is mainly adopted for the analysis and introduction of the observations, collected from an experiment, to make comparisons among the effect of the treatments.

The total variation in any set of numerical observations may be due to a number of causes. These variations may be classified in two groups

(i) Due to assignable causes
(ii) Due to chance causes.

The variations due to assignable causes can be located and measured whereas the variations due to chance causes is beyond the control and therefore cannot be traced separately.

The ANOVA technique is not special to test the significance of two sample variances. Its purpose is to test the equality of several means (mean effect of several treatments) in ANOVA technique the F-test for testing the significance of two measures of variance is applied. These measures of variances differ only if means under consideration are not homogeneous.

Assumption- The analysis of variance (ANOVA) produce is based upon the following two assumptions:

(i) The observation are independent
(ii) Parent population from which the observations has a normal probability distribution
(iii) Various treatment and environment effects are additive in nature.

The components variances are always shown and tested in an analysis of variance table (or ANOVA Table). The ANOVA table consist of five columns as given below, using A,B,C…… as sources of variation. It may be noted that the experimental error is also a source of variation.

Table- The Skelton ANOVA Table

Sources of Variation	Degrees of Freedom (df)	Sum of squares (SS)	Mean Sum squares (SS)	F-values
A B C . . Error				
Total				

The ANOVA enables us to test whether the variance due to a component factor (say A) is significantly more than the variance due to experimental error of not. Here, the mean sum of square due to component is nothing but the variance due to that component. Further, the ratio of component means square error with error mean sum of square (MSE) is distributed as F distribution with corresponding df's.

Basic Principles- We have seen the randomization, replication and local/error control are the three main principles.

The one way ANOVA is used when the field or the experimental material is homogeneous, so there is no need of local control used if there is one source of variation i.e. the treatment. In two way ANOVA, when there are two sources of variations and the three way ANOVA when there are three sources of variation are discussed. These designs take into account the variation in the experimental units. The local control principle plays an important role in these designs.

6.4 ONE WAY ANALYSIS OF VARIANCE

It is very simplest design using the two essential principles as replication and randomization and also known as completely randomized design (CRD). In CRD the units are taken in a single group, as for as possible the units forming the group should be homogeneous.

Let there be k treatment T_1, T_2, ……T_k (or level of a factor) in an experiment. Let the i^{th} treatment be replicate ri times for i= 1,2,3……..k. Thus the total number of experimental units required for the design is n= $\sum r$. In the CRD, we allocate the k treatments completely at random to the n units subject to the condition that the i^{th} treatment appears in r_i units for i=1,2,….k. A particular case of this equal replication for different treatments, where $r_1= r_2= r_3$……rk=r so that n=rk.

Layout- The terms layout refers to the placement of treatments to the experimental units according to the condition of the design. There are several methods for random allocation of treatment to the experimental units; (i) use of random number table, (ii) Lottery system, (iii) Tossing of coin. Suppose we have four treatments A,B,C,D and we have to replicated each treatment four times than the arrangement may be

A	B	C	D
A	C	B	D
C	A	D	B
D	A	B	C

Analysis- This design provides a one way classified data according to level of a single factors treatment. For its analysis following modes is taken

$$y \quad \mu \quad t \quad e \; ; \quad \begin{matrix} i & 1,2,\ldots\ldots\ldots.k \\ j & 1,2,\ldots\ldots..r \end{matrix}$$

Where y_{ij} is the random variable corresponding to the observations obtained from the j_{th} replicate of the i_{th} treatment and e_{ij} is the error component. The error components are independently normally distributed with mean zero and constant variance $\sigma_{,}$, μ is the general effect, t_i is the fixed effect due to the i^{th} treatment. We are interested in hypothesis $H_0=T_i=t_2=\ldots\ldots\ldots\ldots=t_k$ against the alternative hypothesis that t_i are not all equal.

Let $\sum y \quad T$ be the observation total of the i^{th} treatment.

We have treatment sum of the equal $\sum \quad — \quad —$

Where

$$G \quad T, n \quad r$$

Total sum of square

$$y \quad \frac{G}{n}$$

Error sum of square = Total sum of square – treatment sum of squares

The analysis of variance is given below:

ANOVA for one way classified data or completely randomized design

ANOVA: Table of Completely Randomized Designs:

Sources of Variation	Degrees of Freedom (df)	Sum of squares (SS)	Mean Sum squares (s.s./d.f)	F-values
Treatments	k-1	$\frac{T}{r} \quad \frac{G}{n}$	S	$\frac{S}{S}$
Error (within treatment)	n-k	By subtraction	S	
Total	n-1	$Y \quad \frac{G}{n}$		

The hypothesis that treatment have equal are tested by the F-test. If F is not significance, at desired level of significance, the treatments can be considered to have equal effects. If F is significant the treatment effects are not equal. In such

cases it becomes necessary to estimate and test individuals treatment combinations in which experimenter may be interested. Estimate of any treatment contrasts

$\sum \quad l \ y$,where ti denotes the effect of i^{th} treatment and

Where y , —

and

$$V \quad l \ y \quad \sigma \quad \frac{l}{r}$$

Where σ is the error variance which is estimate by the error mean squares, S in the above table. Significance of the contrast can be tested by t-test, where

$$r \quad \frac{|\sum l \ y|}{S \ \sum \frac{l}{r}} \ with \ n \quad k \ d.f.$$

Contrasts of the type (t_i-t_m) in which experiment are often interested are obtained from $\sum l \ t$ by putting l_i=1, l_m=-1 and zero for the other l's.

Merits and Demerits- The CRD is useful in small preliminary experiment and also in certain types of animal or laboratory experiments where the experimental units are homogeneous. There is complete flexibility in the number of treatment and the number of their replications, which may vary from treatment to treatment units or on entire treatment are missing. The CRD provides maximum degree of freedom for the estimation of the experimental error.

The main objection against the CRD is that the principle of local control had not been used in the design. It is seldom used in field experiments because the plots are not homogeneous. It is generally useful in laboratory experiments.

6.5 TWO WAY ANOVA

We have seen that in a completely randomized design no local control measure was adopted as the experimental units are supported to be homogeneous. Usually, when experiments require a large number of experimental units, one way design cannot ensure precision of the estimates of treatment effects. An improvement of the one way design can be obtained by providing error control measures as described below. The resulting design is called two way/randomized block design.

Let there be k treatments. Each of the treatments is replicated the same number of times in this design. Let r denote the number of replications of each treatment. The total number of experimental units is, therefore, kr. These units are arranged into r blocks each of size k. The error control measures in this design consists of making the units in each of blocks homogeneous.

In clinical or similar trials where animals like rats, guinea pigs, etc. are the experimental units, animals coming from the same litter may form blocks. In

general blocks are formed with unites having common characteristics, which are known to have influence on the variate under study.

The number of blocks in the design is the same as the number of replications. The k treatments are allocated at random to the k plots in each block.

Number of treatments = k= Number of plots in a block

Number of blocks=r= Number of replication of a treatment.

This type of homogeneous grouping of the experimental units and the random allocation of the treatments separately in each block are the two main characteristic features of randomized block design. Actual number of replication in the design is determined by the availability of resources and considerations of cost and precision. A method of finding the number of replications from considerations of sensitively of comparison of the treatments has been discussed earlier.

Analysis- The data collected from experiments with randomized block design form a two way classification, that is, classified according to the levels of two factors, viz, block and treatments. There are kr cells in the two way table with one observation in each cell.

We take the model

$$y \quad \mu \quad t \quad e \; ; \quad \begin{matrix} i & 1,2,\ldots\ldots\ldots k \\ j & 1,2,\ldots\ldots\ldots r \end{matrix}$$

Where y denotes the observation on the variable from ith treatment in jth block, μ t b are respectively the general mean, effect of the ith treatment and affect of the jth block as explained earlier. These effects are fixed and e_{ij} is the error component which is random variable. e_{ij}'s are assumed to be normally and independently distributed with zero mean and a constant variance σ .

Following the method of analysis of variance for finding sums of squares due to block treatments and error we proceed as follows:

Let

$$y \quad T \quad i \quad 1,2,\ldots\ldots\ldots k$$

= observation total of ith treatment and

$$y \quad B \quad j \quad 1,2,\ldots\ldots\ldots r$$

= observation total of jth block.

These are the marginal total of the two-way data table.

Further,

$$T \quad B \quad G$$

We shall call G^2/rk as correction factor denoted by C.F.,

So that CF —

Sum of squares due to treatment =$\sum$ — $C.F.$

Sum of squares due to blocks=$\sum$ — $C.F.$

Total sum of squares = $\sum$ y $C.F.$

Table: Analysis of Variance of Two way/Randomized Block Design

Sources of Variation	Degrees of Freedom (d.f.)	Sum of squares (s.s.)	Mean squares M.S.=(s.s./d.f)	F-ratio
Block	r-1	$\frac{B}{k}$ $C.F.$	S $\frac{SSB}{r \quad 1}$	F $\frac{S}{S}$
Treatments	k-1	$\frac{T}{k}$ $C.F.$	S $\frac{SST}{k \quad 1}$ MST	$\frac{MST}{MSE}$
Error	(r-1) (k-1)	By subtraction= SSE		
Total	Rk-1	y $C.F.$		

The null hypothesis H_0 that the treatments have equal effects is tested by F-test.

Under H_0, we have

$$F \quad \frac{MST}{MSE} \sim F \quad ,$$

Test at level of significance α as follows:

Rejected H_0 if F F_0

Accept H_0 if F < F_0

6.6 THREE WAY ANOVA

Two way designs is an improvement over one way designs in the sense that is provides error control measures for the elimination of block variations. This principle can be extended further to improve two way designs by eliminating more sources of variations. Three way/Latin square designs is one such improved

design with provision for the elimination of two sources of variation.

Let there be k treatments, each replicated k times so that the total number of experimental units required is k^2. Let P and Q denote two factors whose variability are to be eliminated from the experimental error by having a suitable design. Evidently both these factors should be related to the variate under study so that their variability may influence the variability of the variate under study. These two are actually the controlled factors. Each of the factors P and Q is taken at k levels. The total number of level combinations of the two factors is k^2. The k^2 experimental units are now so chosen that each unit possesses a different level combination of the two factors.

Originally Latin square designs were defined for eliminating the variation of two factors which are generally called row and column. Though it is necessary that the two factors should always be called row and column it has become customary to define latin square by calling the factors as row and column. The experimental units are obtained as specified above the treatments are allotted to these units in the following way; the k treatments are allotted to the k2 units in such a manner that each treatment occurs only once in each level of the factor P and once in each level of the factor Q. This requires that each treatment should be replicated k times.

If a two-ways table is formed with the levels of the factors P and Q such that the levels of P denote the rows and the level of Q denote the column, then the Latin square designs requires that the treatments should be so allotted to the k^2 cells of this table that each treatment occurs once in each row and once in each column. Such an arrangement is called a latin square of order k.

For example suppose there are five treatments denoted by A,B,C, D and E. Then the following arrangements in a 5 5 square is a latin square design.

	Level of P				
Level of Q	P_1	P_2	P_3	P_4	P_5
q_1	A	B	C	D	E
q_2	D	E	A	B	C
q_3	B	C	D	E	A
q_4	E	A	B	C	D
q_5	C	D	E	A	B

It has been pointed out earlier that this design is effective if the two factors P and Q can cause variability in the variate under study. If one of the factors does not have substantial influence on the variate under study, elimination of its variance may not reduce the error variance. In such a situation a latin squre design is no improvement over the randomized block designs. So, unless it is known that both the factors cause sufficient variation in the variate under study, it is better to adopt a randomized block design. In agriculture experiment if there is soil fertility

in two mutually perpendicular directors, then the adoption of a lain square design with rows and column along the director of fertility gradients proves useful.

The LSD is an incomplete three layouts, where three factors namely, row, column and treatment, are at the same number of levels (k). From the layout, it is obvious that in LSD there is need of k^2 experimental units (plots). But for a complete three ways layout with each factor at k level one needs m3 units. Hence there is saving of (k^3-k^2) experimental units in using LSD.

Analysis of L.S.D.- In latin square designs there are three factors. These are the factors P,Q and treatments. The data collected from this design are, therefore analyzed as a three way classified data. Actually, there should have been k^3 observations as there are three factors each at k level. But because of the particular allocation of treatments to the cell, there is only one observation per cell instead of k in the usual three way classified orthogonal data. As a result we can obtain only the sums of squares due to each of the three factors and error sum of squares. None of the interaction sums of squares of the factors can be obtained accordingly, we take the model.

$$y \quad \mu \quad r \quad c \quad t \quad e$$

Where y denotes the observation on the variable of interest in j[th] row j[th] column and under s[th] treatment $\mu, r, c, t \quad i, j, s \quad 1,2, \ldots . k$ are fixed effects denoting in order the general mean, the row, the column and the treatment effects. The e is the error component assumed to be independently and normally distributed with zero mean and a constant variance.

The analysis is conducted by following a similar procedure as described for the analysis of two way classified data. The different rums of squares are obtained as below:

Let the data be arranged first in a row x column table such that □□□ denotes the observation of (i,j)[th] cell of the table.

Let Ri = $\sum y$ = i[th] row total (i=1,2,….k).

$C \quad \sum y$ = j[th] row total (j=1,2,….k).

T_S = sum of those observations which come from sth treatment

= sth treatment observation totl (s=1,2,…..k).

$G \quad \sum R$ =grand total

Correction Factors CF —

Treatment sum of squares =$\sum$ — $C.F.$

Row sum of squares =$\sum$ — $C.F.$

Column sum of squares = $\sum$ — $C.F.$

Table: Analysis of Variance of Completely Randomized Designs

Sources of Variation	d.f.	S.S.	M.S.= (s.s./d.f)	F
Row	k-1	$\frac{R}{k}$ $C.F.$ $S.S.R$	S	
Column	k-1	$\frac{C}{k}$ $C.F.$ $S.S.C$	S	
Treatments	K-1	$\frac{T}{k}$ $C.F.$ SST	S	$\frac{S}{S}$
Error	(k-1)(k-2)	By subtraction= SSE		
Total	K^2-1	y $C.F.$ TSS		

The hypothesis of equal treatment is tested by F-test where F is the ratio of treatment mean squares to error mean squares. If F is not significant, treatment effects do not differ significantly among themselves. If F is significant further studies to test the significance of any treatment contrast can be made in exactly the same way as discussed for randomized block designs.

6.7 SUMMERY

In this unit we have studied about ANOVA and the analysis of one way, two way and three way classified data.

6.8 CHECK YOUR PROGRESS

1. Four groups of patients were subjected to the different treatments for a particular disease. At the end of a specified time period each was given a test to measure treatment effectiveness. the scores were obtained. Conduct appropriate test to find, if there is any significant difference in the treatments. Also give the ANOVA table.

A	B	C	D
60	76	58	95

88	70	74	90
70	80	66	80
80	90	60	87
76	75	82	88
70	79	75	85

Conduct appropriate test to find, if there is any significant difference in the treatment. ($F = -2.6$, $F_a = 4.26$)

2. The following table gives the gains in weights of four different types of pigs fed on three different ratios. Test whether the rations or the pig types differ in their on mean weight.

Types of pigs

Types of Rations		I	II	III	IV
	I	7	16	10	11
	II	15	14	15	14
	III	8	16	7	11

6.9 FURTHER READINGS

1. Alok Dey (1986): Theory of Block Designs Wiley Eastern.
2. Das M.N. and Giri N. (1979): Design and analysis of Experiments.
3. Joshi D.D. (1987): Linear Estimation and Design of Experiments Wiley Eastern.
4. Goon, Gupta & Dasgupta : Fundamentals of Statistics Vol. I and Vol. II The World Press Pvt. Ltd., Kolkata.

NOTES